Approximation of Profit-and-Loss Distributions
(Part II)

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Summary: Part I [13] introduces the application of the barycentric approximation methodology for evaluating profit-and-loss distributions numerically. Although, convergence of the quantiles is ensured by the weak convergence of the discrete measures, as proclaimed in [13], recent numerical results have indicated that the approximations of the profit-and-loss distribution are less practical when the portfolio gets a reasonable complexity. This experience has revealed that the *weak convergence* of the probability measures appears not to be strong enough for evaluating quantiles numerically in a satisfactory way.

Thereupon, the authors have focused on information offered by the barycentric approximation but still unused in the algorithmic procedure [13]. It has been realized that the dual to the derived discrete probability measure helps evaluate the profit-and-loss distribution in a better way. In this Part II, the barycentric approximation technique is outlined and benchmarked with the intention to focus on the dual viewpoint for simplicial refinement. This technique poses no assumption on the risk factor space, except that the variance-covariance matrix of the risk factors exist. Therefore, it is applicable for general multivariate or empirical distributions. Furthermore, the technique provides approximation of the risk profile as well as of the risk factor distribution.

Beforehand, various test environments are specified which help illustrate the sensitivity of value-at-risk numbers. These environments are characterized by the probability measure P of the risk factors and a risk profile g which represents the payoff structure of some portfolio. The corresponding numerical results illustrate the sensitivity of value-at-risk with respect to market volatility and correlation of risk factors. This provides information on the model risk one is exposed to within the value-at-risk approach.

1 Introduction

The evaluation of a portfolio and its risk exposure with respect to market movements become difficult as soon as contingent claims are involved. In case only the performance of a portfolio has to be determined, practitioners use the mark-to-market pricing each trading day and observe the value changes of the underlying portfolio ex post. A price series becomes available which reveals not only the performance but also the *risk-return pattern* of the various financial activities untertaken by the portfolio manager. Practitioners prefer this approach due to its simplicity, there is no need for information ex ante, neither for determining the value functions of the financial instruments nor for assessing the probability distribution of the risk factors. However, it is this lack of information, that makes it difficult to rebalance the portfolio for achieving an improved risk-return pattern in an efficient way.

The risk-return pattern

Practitioners often identify the *risk-return* pattern of financial instruments through the average return and the volatility of the return. In this work, we will characterize the risk-return pattern of a portfolio with the so-called *profit-and-loss distribution*, associated with a specified planning horizon. Clearly, the profit-and-loss distribution is determined by the value functions of the instruments and the probability distribution of the risk factors. The latter represents the dynamics of the risk factors up to the end of the prespecified holding period. Herein, the notions *risk-return pattern* and *profit-and-loss distribution* are used synonymously.

Requesting an appealing approximation of the profit-and-loss distribution requires to determine the *dynamics of the risk factors* and *the value functions* of the corresponding contingent claims held in the portfolio.

The *dynamics of risk factors* are commonly modelled via normal or lognormal distributions. However, it has been observed empirically that market movements are distributed non-normally, respectively non-lognormally. For taking this into consideration one may think of employing series of high frequency data of the risk factors, which help assess more general distributions for the market movements.

The *value function* of contingent claims are often given implicitly through partial differential equations, which have to be solved. The most common approach still used for valuing derivatives is based on the Black-Scholes model, which allows for solving the underlying partial differential equation analytically. In recent years limited applicability of the Black-Scholes model has been seen by both scientists and practitioners due to the fact that the volatility is supposed to be deterministic and known beforehand; in addition, empirically observed characteristics of certain risk factors, like the mean reverting property of interest rates, cannot be incorporated adequately. In finance, various extensions of the Black-Scholes approach and new valuation models have been developed which take into account various dynamic features for pricing contingent claims in a more realistic manner (see e.g. Hull 1997 [18], Wilmott et al. 1994 [35]). Therefore these models build the basis for determining appealing approximations for the value functions, and, finally, for the desired risk-return pattern.

Practical viewpoints

The practice of risk management faces analytical and organizational problems which make the pointwise evaluation of the profit-and-loss distribution onerous.

Analytical problems: The progress of financial engineering has created increasingly sophisticated financial instruments, for which there is no analytically closed-form solution to their value functions. Instead, these value functions are given implicitly, so that even pointwise evaluations of the profit-and-loss distribution become difficult.

Organization problems: Risk management units of large financial companies have to manage a considerable quantity of data. The portfolios of such companies likely contain thousands of financial instruments which depend on hundreds of risk factors. The delocalization of the trading units of a worldwide institution causes the portfolio to be traded continuously, which results in permanent shifts of the portfolio structure. This induces permanent changes of the underlying risk factors and dynamic changes of the risk-return pattern.

Assessing the risk-return pattern of a portfolio provides the portfolio manager with information on the frequency and amount of both, potential loss and potential profit. In case of linear or quadratic value functions and normally distributed risk factors the quantiles of the profit-and-loss distribution are available pointwise, i.e. with respect to prespecified levels. The challenge lies in the numerical evaluation of quantiles which becomes onerous in case the nonlinear value function of a portfolio is given implicitly and the risk factors are distributed non-normally. In practice, quantiles which represent a loss are also called *value-at-risk*. Herein, *value-at-risk* and *quantiles of a profit-and-loss distribution* are used synonymously.

Coherent risk-measures

Any number which represents the potential loss of a portfolio in an adequate predefined sense may be accepted as a *risk measure*. In Artzner et al. 1996 [1] a distinguished class of risk measures has been introduced, the so-called *coherent risk measures*. These pay attention only to those market movements that cause a loss to the portfolio manager. Those market movements which provide profits are not taken into account. Coherent risk measures are not based on market expectations of individual portfolio managers. Instead, a distinguished set of risk factor distributions characterize the coherent risk measure and allows for identifying the potential loss. The elements of this set are called *generalized scenarios* in Artzner et al. [1]. It should be noted that coherent risk measures remain unchanged in case the set of generalized scenarios is convexified. In this sense coherent risk measures help regulators assess the capital requirement with respect to a distinguished set of market dynamics for controlling the possibility of bankruptcy.

One common coherent risk measure is the *maximum loss* derived with respect to a predefined feasible region of market movements. The distinguished set of probability measures, which characterizes this measure in the sense of Artzner et al. [1], consists of the one-point distributions at each point of a confidence region and its convexification. The challenge for evaluating the maximum loss lies in the minimization of a nonconvex, high-dimensional value function. For details on the numerical solvability of the *maximum loss approach* it is referred to Studer and Lüthi 1995 and 1997 [32, 33, 34].

Contents of this paper

We focus on algorithmic procedures for determining the risk-return pattern of a portfolio. As mentioned above, the risk-return pattern characterized by the profit-and-loss distribution is completely determined by the value functions of the underlying instruments and by the risk factor distribution. For the reason of adequate benchmarking, we focus on normal distributed risk factors and on value functions which stem from the Black-Scholes model. It will become clear below for which methodologies these assumptions may be relaxed in what way. This work is seen as a contribution for helping develop and improve risk assessement tools for both trading and management.

The structure of this work is as follows: In section 2 the problem statement is introduced formally. A specific financial instrument having been issued by a Swiss bank in June 97 is taken as an example for illustrating the profit-and-loss distribution. Section 3 roughly surveys existing approaches. In particular, the goodness and the numerical effort of the Delta approximation, the Delta-Gamma approximation, the Monte Carlo simulation and the historical simulation are discussed. In Section 4 various test environments are specified which help illustrate the sensitivity of value-at-risk numbers. These environments are characterized by the probability measure P of the risk factors and a risk profile g which represents the payoff structure of some portfolio. Section 5 reports on the numerical results within the specified environments illustrating the sensitivity of value-at-risk with respect to market volatility and correlation of risk factors. This provides information on the model risk one is exposed to within the value-at-risk approach. Section 6 reveals that the dual to the derived discrete probability measure helps evaluate the profit-and-loss distribution in an appealing way. The barycentric approximation technique is outlined with the focus on this dual viewpoint for simplicial refinement. Section 7 benchmarks the barycentric approximation to the Delta-Gamma approximation for predefined FX-portfolios and for the ROE Warrant on the ABB stock. Section 8 concludes and provides an outlook for future research activities which should help improve an active portfolio management with the value-at-risk approach.

2 Problem statement

Let $\omega = (\omega_1, \ldots, \omega_M) \in \mathbb{R}^M$ denote the changes of M risk factors which define the value $g(\omega)$ of an underlying portfolio. $g(\cdot)$ represents a real-valued function from a proper domain $\Omega \subset \mathbb{R}^M$ to $\mathbb{R}$ and is called *value function* or *risk profile*. Let be $g(0) = 0$ and denote with $-g(\cdot)$ the short position of the underlying portfolio. Clearly, the investor who sells the portfolio has risk profile $-g$. The outcome of ω at the end of the holding period $[0, 1]$ is given by its probability measure P on the Borel space $(\mathbb{R}^M, \mathcal{B})$. The profit-and-loss distribution (i.e. the risk-return pattern) is then given by the induced probability measure P_g of the transformed random variable $g(\omega)$ on $(\mathbb{R}, \mathcal{B})$. The associated distribution function F_g is given through

$$F_g(\hat{v}) = P\{\omega | g(\omega) \leq \hat{v}\}. \tag{1}$$

It is stressed that only the frequency, that the portfolio value exceeds $\hat{v}$, is revealed. There is no information available concerning to what extent the portfolio value does not come up to $\hat{v}$. This information would become available from the lower partial moments of order ≥ 1. Numerical evaluations are more cumbersome due to the inherent additional transformation of the implicitly given risk profile. This will not be discussed in this work but postponed to future research activities.

The positions in the portfolio are supposed to remain fixed within a pregiven holding period. Due to its characterization a *coherent risk measure* provides neither information on the frequency (i.e. probability) of the potential loss nor on the frequency and amount of potential profits. Hence, no information can be deduced for the risk-return pattern and its asymmetry, which provides the basis for improving the portfolio management. This is the motivation for focusing on the profit-and-loss distribution. It is stressed that quantiles if accepted as risk measures do not fulfill the subadditivity condition and, hence, are not coherent. Evaluating quantiles requires that a unique probability measure is used for modeling the market movements. This unique probability measure is denoted P and may be represented by the martingale measure or by the individial investor's expectations of future market movements.

Let $\hat{v}^{+,\alpha}$ be the *quantiles* or *value-at-risk numbers (VaR)* of level α for the long portfolio g and $\hat{v}^{-,\alpha}$for the short portfolio $-g$. For the ease of exposition, it is supposed that these quantiles $\hat{v}^{+,\alpha}$, $\hat{v}^{-,\alpha}$ exist and that the distribution functions F_g, F_{-g} are continuous on $\hat{v}^{+,\alpha}$, $\hat{v}^{-,\alpha}$. In case that both refer to the same prespecified level α, i.e. if

$$F_g(\hat{v}^{+,\alpha}) = F_{-g}(\hat{v}^{-,\alpha}) = \alpha, \tag{2}$$

then both values, $\hat{v}^{+,\alpha}$ and $\hat{v}^{-,\alpha}$ reveal information on the asymmetry of the risk-return pattern.

Some special cases, in which the profit-and-loss distribution F_g can be represented by some standard distribution, will be discussed below roughly.

For the ease of understanding the profit-and-loss distribution of a warrant, which has been issued recently by a Swiss private bank (see [4]), is evaluated. This *ROE warrant on ABB* is seen as an alternative to money-market instruments. The warrant expires on June 24, 1998 and has been priced with SFr. 1875.- on June 15, 1997. At this date the price of the ABB stock has been SFr. 2130.-. Depending on the ABB stock price at the expiration date two payments are possible: i) in case the price is greater or equal the cap of SFr. 2100.- then the holder of one warrant obtains the cap of Sfr. 2100.- as payment; ii) in case the ABB stock price closes below that cap SFr. 2100.- then the holder of one warrant receives one ABB stock.

We are interested in the value-at-risk corresponding to level 1%, 3% and 5%. The time horizon is 3 months. The two-dimensional risk profile of the warrant is illustrated in Figure 1. The

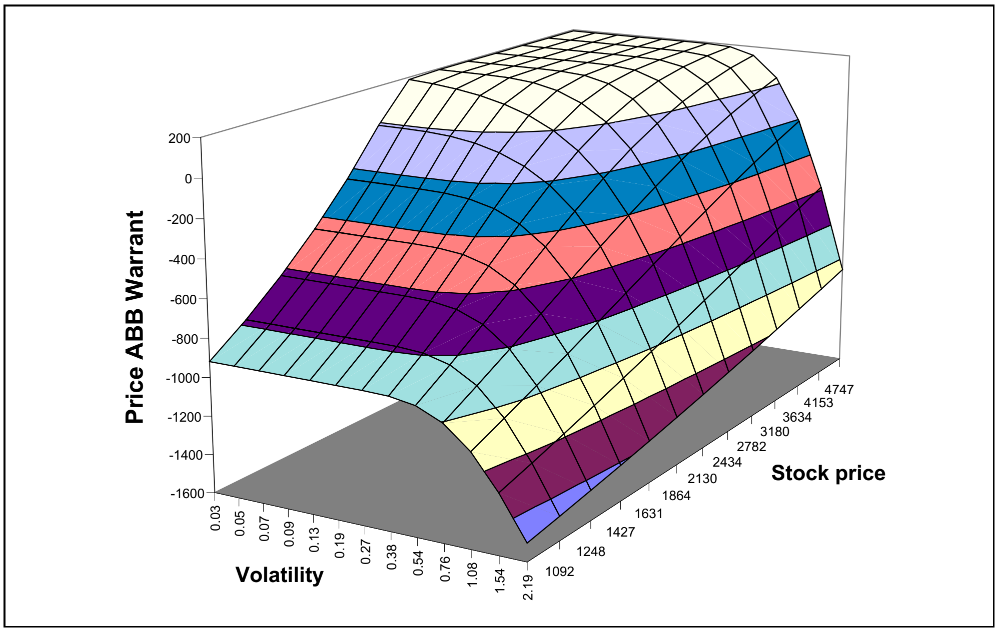

Figure 1: Risk profile of the ROE warrant on ABB

current stock price p_0 is Sfr. 2130.- with a volatility σ_0 set to 26%. The risk factor changes of price ω_1 and volatility ω_2 are normally distributed $N(\mu, \Sigma)$ with parameters

$$\mu = 0 \in \mathbb{R}^2 \qquad \Sigma^{ABB} := \begin{pmatrix} 0.070756 & 0.037240 \\ 0.037240 & 0.490000 \end{pmatrix}.$$

The associated profit-and-loss distribution is continuous and shown in Figure 2. The value-

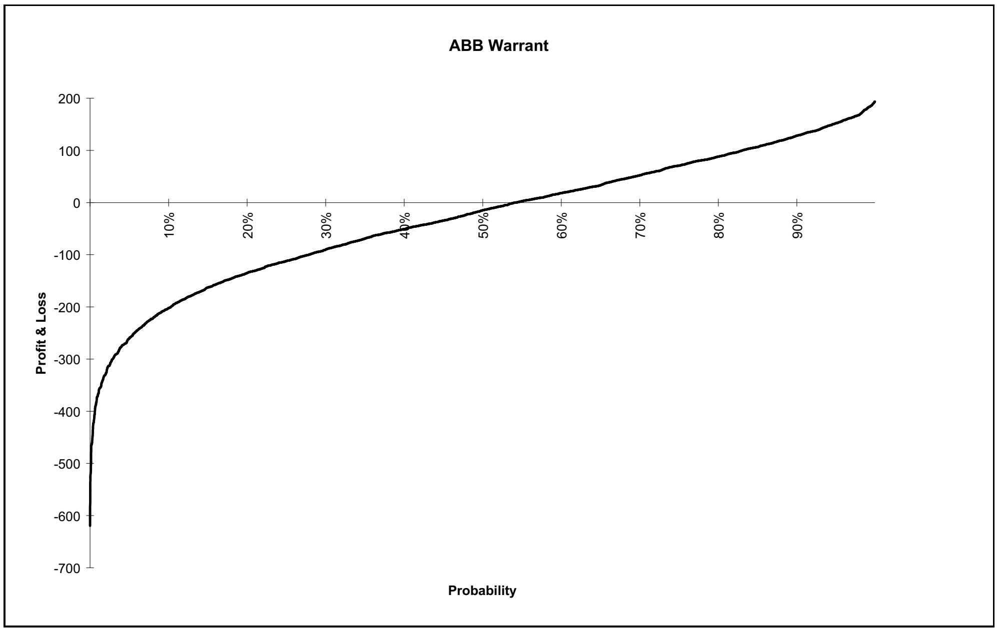

Figure 2: Profit-and-loss distribution of the ROE warrant on ABB

at-risk numbers of levels 1%, 3% and 5% are listed in Table 1 and reveal information on the asymmetry of the risk-return pattern.

α	$v^{+,\alpha}$	$v^{-,\alpha}$
1%	378.123	180.683
3%	300.441	163.559
5%	261.284	151.669

Table 1: VaR for the ROE Warrant on ABB

3 Existing Approaches

We briefly review existing value-at-risk methodologies for measuring the potential loss of a portfolio risk profile $g(\omega)$ with respect to a corresponding domain $\Omega \subset \mathbb{R}^M$. Goodness of the value-at-risk proxies and their associated numerical effort is discussed. It will become clear from the arguments below that each approach offers valuable information on the risk exposure. However, each has to be utilized with care.

Delta approximation of the risk-profile

Linear approximations of the value functions at current price levels are denoted *Delta approximations*. These are widely used in classical risk management, and are known as *duration analysis* in bond management or *Delta hedging* in portfolio management (see [8, 14, 18, 25]). Substituting the risk profile by linear functions locally helps overcome the difficulty of implicitly given value functions and provide analytical ways for determining the value-at-risk in case the risk factors are normally distributed.

Mostly, the instanteneous first order sensitivities $\Delta \in \mathbb{R}^M$ of the value functions $g(\cdot)$ with respect to the risk factors $\omega \in \mathbb{R}^M$ - the socalled *Delta* - determines the linear approximation $\hat{g}(\cdot) = \Delta'\omega$ at $\omega = 0$. In case that the risk factors $\omega \in \mathbb{R}^M$ are distributed according to $N(0, \Sigma)$ $\hat{g}(\omega) \in \mathbb{R}$ is distributed according to $N(0, \Delta'\Sigma\Delta)$ which represents a symmetric risk-return pattern. Hence, the value-at-risk numbers associated with $\hat{g}(\omega)$ and $-\hat{g}(\omega)$ respectively, are given analytically by

$$\hat{v}_{\Delta}^{-,\alpha} = \hat{v}_{\Delta}^{+,\alpha} = c_\alpha \cdot \Delta'\Sigma\Delta \tag{3}$$

and serve as a proxy for the value-at-risk of g and of $-g$, respectively. The coefficient c_α corresponds to level α, i.e. $c_\alpha = 1.64$ for $\alpha = 5\%$, $c_\alpha = 1.88$ for $\alpha = 3\%$, and $c_\alpha = 2.33$ for $\alpha = 1\%$. Clearly, their goodness depends on the degree of nonlinearity of the risk profile on Ω. It should be stressed that the goodness of the linear approximation decreases with increasing holding period if options are included.

In case the Δ of a portfolio is close to $0 \in \mathbb{R}^M$ the portfolio is called *Delta-hedged.* The value of a Delta-hedged portfolio remains unchanged for small changes in the risk factor, the value-at-risk is close to 0. However, it might have severe impact on the portfolio value if ω leaves some neighbourhood of 0.

In case instantaneous first-order sensitivities are used, i.e. the Δ at $\omega = 0$, the numerical effort is restricted to M evaluations of first order derivatives. In many cases, it is useful to work with a Δ that corresponds to the sensitivities of g at some nonzero risk factors $\omega \in \Omega$. This makes an additional evaluation of the portfolio at $\omega \neq 0$ necessary to obtain a linear affine approximation $\hat{g} = \Delta'\omega + \text{const}$. Taking an average of first order sensitivities of g on Ω may yield better linearizations but requires additional effort for selecting a number of distinguished risk factor values at which the portfolio and its first order sensitivities are evaluated. It is noted that better linearizations need not necessarily result in better value-at-risk approximations.

Applying the Delta approximation to the ROE warrant yields the risk profile

$$\hat{g}(\omega) = 0.596098 w_1 - 828.2664 w_2.$$

The variance-covarince matrix of the logarithms of the risk factors is given with Σ^{ABB}. The associated value-at-risk numbers $\hat{v}^{+,\alpha}, \hat{v}_{\Delta}^{+,\alpha} \hat{v}^{-,\alpha}$ and $\hat{v}_{\Delta}^{-,\alpha}$ are listed in Table 2. These results illustrate that the accuracy is insufficient and the severe asymmetric shape of the true profit-and-loss distribution is not mapped adequately.

α	$v^{+,\alpha}$	$v_{\Delta}^{+,\alpha}$	$v^{-,\alpha}$	$v_{\Delta}^{-,\alpha}$
1%	378.123	289.425	180.683	315.600
3%	300.441	230.891	163.559	251.365
5%	261.284	199.966	151.669	212.613

Table 2: Delta approximation of VaR for the ROE warrant

Delta-Gamma approximation of the risk profile

To incorporate the nonlinearity of a risk profile second-order information of g on Ω is used. Substituting the risk profile locally by a quadratic function $\tilde{g}$ helps overcome the difficulty of implicitly given value functions g and provides additional information on the curvature of g, i.e. on the sensitivities of Δ. Normally distributed risk factors allow for deriving the value-at-risk numbers of $\tilde{g}$ analytically, which serve as proxies for the corresponding value-at-risk numbers of g. Formally,

$$\tilde{g}(\omega) = \omega'\Gamma\omega + \Delta'\omega, \tag{4}$$

where $\Gamma \in \mathbb{R}^{M\times M}$ represents the Hessian of g at some point $\omega \in \Omega$, i.e. the matrix of the second-order derivatives of g. As above, Δ represents the M-dimensional vector of first-order sensitivities. In literature, $\tilde{g}$ is known as the *Delta-Gamma approximation.* Obviously, $\tilde{g}$ is no longer distributed normally. However, as shown in Rouvinez 1997[27], the distribution $F_{\tilde{g}}$ of $\tilde{g}$ is representable as a combination of non-central χ^2-distributions, whose corresponding quantiles are given in analytical form.

The numerical effort for determinig quadratic approximations of g increases with order 2 in the dimension M. This is due to evaluating $\frac{M(M+1)}{2}$ items of Γ. If Γ represents a kind of average of Hessian taken locally at various points $\omega \in \Omega$ then the effort is a multiple. Again, better quadratic approximations need not necessarily result in better value-at-risk estimates. Which Hessian Γ should be selected, so that the value-at-risk of $\tilde{g}$ serves as sufficient proxy for the value-at-risk of g, is of practical importance.

JP Morgan [20] has analysed the goodness of its own Delta-Gamma approximation for the pricing of call and put options. The widely used Black and Scholes formula serves as a benchmark. The results show that the relative error is dependent on the relation of the spot and strike price and on the time to maturity. The error increases when the option approaches expiration at-the-money. An obvious explanation is offered by the nondifferentiability of the risk profile at the strike price when the option expires.

Applying the Delta-Gamma approximation to the ROE warrant yields a quadratic approximation $\tilde{g}$ for the risk profile

$$\tilde{g}(\omega) = 0.596098w_1 - 828.2664w_2 - 0.5(0.000681w_1^2 + 0.0693118w_1w_2 - 18.02847w_2^2)$$

The associated value-at-risk numbers $\hat{v}^{+,\alpha}, \hat{v}_{\Delta-\Gamma}^{+,\alpha}, \hat{v}^{-,\alpha}$ and $\hat{v}_{\Delta-\Gamma}^{-,\alpha}$ are listed in Table 3. These results illustrate sufficient accuracy and an adequate mapping of the asymmetric profit-and-loss distribution.

α	$v^{+,\alpha}$	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}$	$v^{-,\alpha}_{\Delta-\Gamma}$
1%	378.123	369.296	180.683	195.977
3%	300.441	293.088	163.559	166.341
5%	261.284	253.719	151.669	150.037

Table 3: Delta-Gamma approximation of the VaR for the ROE warrant

Monte Carlo simulation

The probability measure P on the M-dimensional risk factor space $(\mathbb{R}^M, \mathcal{B})$ associated with a fixed holding period $[0, T]$ is supposed to be known. The values ω^i $(i = 1, \ldots, N)$ represent the risk factor movements up to time T and are drawn randomly with size N by a random number generator, which maps the probability measure P adequately. It is noted that P need not necessarily be of normal type. The underlying portfolio is priced for each of the N randomly generated ω^i, i.e. the value function g is evaluated at each ω^i. The *Monte Carlo simulation* (see [10, 28]) yields an empirical distribution F_g^e of g and, hence, an approximation of the real profit-and-loss distribution F_g. The quantiles of F_g^e are proxies for the quantiles of F_g.

The applicability of the Monte-Carlo simulation is limited due to the fact that the mapping of both the probability measure P and the risk profile g is adequate only for a large sample size N, say for $N \geq 10'000$. On the other hand, the number of portfolio evaluations that can be afforded lie in the hundreds. Monte-Carlo simulation or modern versions, like the quasi-random Monte Carlo simulations (see [9, 19]), is therefore used in practice with care. It should be noted, that the goodness of random number generators impacts the goodness of the value-at-risk numbers. Numerical tests with using various random number generators (see Härtel [17]) have indicated that the variability of the quantiles of the empirical distribution F_g^e taken with respect to various generators is between $1-5\%$. Therefore the variability of the value-at-risk numbers with respect to different generators may be accepted as negligible, at least at this stage.

Historical simulation

In the *historical simulation* (see [29, 30]) the portfolio is evaluated with respect to M-dimensional risk factor movements of the past. This yields an empirical distribution F_g^h of the portfolio changes g which serves as approximation for F_g. Evaluating the skewness and kurtosis of the historical data of each of the M components illustrates whether the normal distribution of the risk factors is valid. This allows conclusions on the goodness of the Delta approximation and the Delta-Gamma approximation at least *ex-post*.

Again, the applicability of the historical simulation is limited by its size N. The length of the past period considered is a trade-off between the sample size and the representativeness of the data. It is capturing possible fat tails but also outliers of the distribution. One has to be aware that the past observations map the future risk factor dynamics. Hence, the goodness of the so derived value-at-risk numbers depends on how accurate the future risk factor movements obey the past movements probabilistically.

In practice, the daily returns are often used as an empirical distribution although the underlying portfolios are modified in the daily business. In this case, the empirical distribution reveals neither information on the riskiness or on the risk-return pattern of the current portfolios, nor can this be utilized for improving the risk-return pattern and, hence, for improving the performance of a portfolio manager. The daily returns do reveal information on the risk attitude of the portfolio manager if its volatility is benchmarked to that of indices.

4 Test Environment

The quantiles of the profit and loss distribution F depend on the probability measure P of a measurable risk factor space $(\Omega, \mathcal{B})$ and on the risk profile $g : \Omega \subset \mathbb{R}^M \to \mathbb{R}$ of the underlying portfolio. The quantiles of F_g represent the value-at-risk (VaR) with respect to a predefined level α. Whether this value-at-risk number reflects the real risk exposure of the portfolio in the prespecified sense depends on how good P maps the future risk factor movements up to the end of the holding period, and on how good g maps the market prices. How to model the risk factor dynamics and the pricing mechanism is of interest for scientists and practitioners. To clarify the contribution of this work, the issue is the evaluation of the quantiles of F_g given the measure P and the risk profile g. Due to the complexity of the problem the quantiles are to be determined not analytically but numerically with some level of inaccuracy. An efficient algorithmic procedure should behave reasonably fast and accurate. The sensitivity of the quantiles with respect to the parameters of market dynamics is one indicator for the model risk. Key parameters for modelling the dynamics are the volatility and the correlation structure of the risk factors. Accurate estimates of these parameters are of capital importance for the goodness of value-at-risk numbers. For recent works on parameter estimation it is referred to Göing ([15, 16]) and the references therein.

In this section various test environments are specified which help illustrate the sensitivity of value-at-risk with respect to volatility and correlation in the subsequent section. These environments are characterized by the probability measure P of the risk factors and an underlying risk profile g which represents the payoff structure of some underlying portfolio.

Environment I)

For tutorial purposes one-dimensional quadratic risk profiles $g^0(\omega), g^+(\omega)$ and $g^-(\omega)$ are considered with ω distributed according to $N(0, \sigma = 1)$ and $N(0, \sigma = 1.3)$. It is assumed that $g^0(\omega), g^+(\omega)$ and $g^-(\omega)$ are nonnegative for $\omega < 0$ and take the value

$$g^0(\omega) = -25\omega, \qquad g^-(\omega) = \frac{-25}{4}\omega^2 \qquad g^+(\omega) = \frac{25}{4}\omega^2 - 50\omega \tag{5}$$

for $\omega \geq 0$. The curvature Γ and slopes Δ are given for $\omega \in [0, 4]$ according to

$$\Gamma^0 = 0 \qquad \Gamma^- = \frac{-50}{4} \qquad \Gamma^+ = \frac{50}{4} \tag{6}$$

and

$$\Delta^0 = -25 \qquad \Delta^- = \frac{-50\omega}{4} \qquad \Delta^+ = \frac{50\omega}{4} - 50. \tag{7}$$

Note that the Delta of g^- is small and close to 0 for small $\omega > 0$, the Delta of g^+ is close to -50 for small $\omega > 0$. The maximum loss of all three profiles subject to $\omega \in [-4, 4]$ is equal to 100.

The curvature reveals the degree of nonlinearity of the risk profile. The graphics below show the risk profiles on the loss region $\omega \in [0, 4]$.

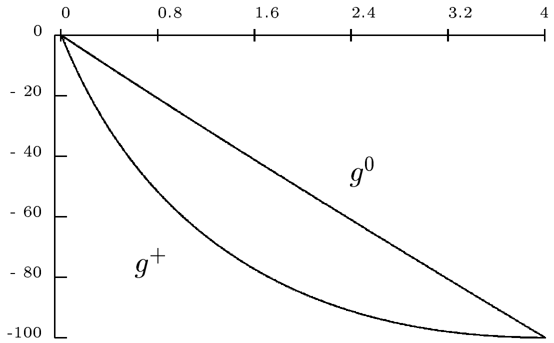

Figure 3: Risk profiles g^+ and g^0

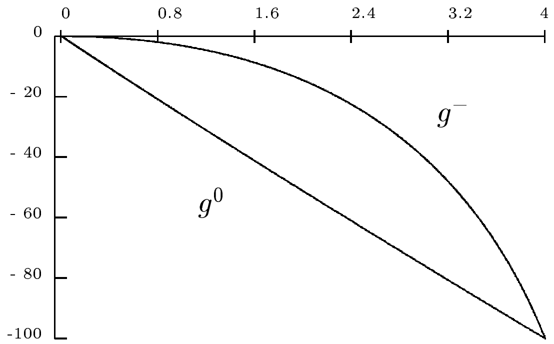

Figure 4: Risk profiles g^- and g^0

Environment II

16− and 4−dimensional linear-quadratic risk profiles are considered with the intention to examine the sensitivity of the value-at-risk with respect to the curvature of a quadratic risk profile and the correlation structure Σ^{II} of the risk factors. The risk factors are supposed to be normally distributed according to $N(0, \Sigma^{II})$. Let the curvature of the risk profile $g(\omega)$ be given by the diagonal matrix D. The scattering of this distribution is characterized by the eigenvectors and eigenvalues of Σ^{II}. The eigenvalues represent the scale of the scattering in direction of the corresponding eigenvector (see Figure 5).

The value-at-risk appears to be more sensitive with respect to the curvature in the direction of the eigenvector the larger the eigenvalue is. In order to make the sensitivities subject to different degrees of correlation comparable, one has to change the curvature of the price function in that direction. Hence, the coordinate system is transformed by the matrix of eigenvectors V^{II} that correspond to an individual correlation structure Σ^{II}. Let the diagonal elements of D^{II} be eigenvalues which represent the desired curvature of a risk-profile along the eigenvectors.

Transforming D^{II} accordingly yields the Hessian matrix $\Gamma^{II} = V^{-1}DV$ (see Figure 6) of the desired quadratic risk profile

$$g^{II}(\omega) = \omega'\Gamma^{II}\omega + \omega'\Delta^{II}. \tag{8}$$

Evaluations have been performed for different degrees of correlation and for different Hessian. In particular, we have considered the uncorrelated case, Σ_u^{II}, a medium correlated case, Σ_m^{II}, and a highly correlated case, Σ_h^{II} (see Appendix). The eigenvector matrices V_u^{II}, V_m^{II} and V_h^{II} are accordingly determined. The diagonal matrices $D_a = -4I$[a] and $D_b = -32I$ yield Hessian matrices $\Gamma_{u,a}^{II}, \Gamma_{m,a}^{II}, \Gamma_{h,a}^{II}, \Gamma_{u,b}^{II}, \Gamma_{m,b}^{II}$ and $\Gamma_{h,b}^{II}$ for the associated risk profiles $g_{u,a}^{II}, g_{m,a}^{II}, g_{h,a}^{II}, g_{u,b}^{II}, g_{m,b}^{II}$ and $g_{h,b}^{II}$:

$$g_{\cdot,\cdot}^{II} = \omega'\Gamma_{\cdot,\cdot}^{II}\omega + \omega'\Delta^{II}. \tag{9}$$

Setting $\Delta^{II} := 16\mathbb{1}$[b] offers high and low curvature relative to the slope (see Appendix).

The 4-dimensional profile and its correlation structure is indexed with $\hat{I}I$. The associated variance-covariance matrices are given by

$$\Sigma_u^{\hat{I}I} := \begin{pmatrix} 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 \end{pmatrix}$$

$$\Sigma_m^{\hat{I}I} := \begin{pmatrix} 1.000 & 0.049 & 0.387 & 0.074 \\ 0.049 & 1.000 & 0.477 & 0.309 \\ 0.387 & 0.477 & 1.000 & 0.227 \\ 0.074 & 0.309 & 0.227 & 1.000 \end{pmatrix}$$

$$\Sigma_h^{\hat{I}I} := \begin{pmatrix} 1.000 & 0.755 & 0.855 & 0.811 \\ 0.755 & 1.000 & 0.884 & 0.957 \\ 0.855 & 0.884 & 1.000 & 0.911 \\ 0.811 & 0.957 & 0.911 & 1.000 \end{pmatrix}$$

and represent submatrices of $\Sigma_u^{II}, \Sigma_m^{II}$ and Σ_h^{II}. The eigenvector matrices $V_u^{\hat{I}I}$, $V_m^{\hat{I}I}$ and $V_h^{\hat{I}I}$ are determined accordingly. The diagonal matrices $D_a = -4I$ and $D_b = -32I$ yield Hessian matrices $\Gamma_{u,a}^{\hat{I}I}, \Gamma_{m,a}^{\hat{I}I}, \Gamma_{h,a}^{\hat{I}I}, \Gamma_{u,b}^{\hat{I}I}, \Gamma_{m,b}^{\hat{I}I}$ and $\Gamma_{h,b}^{\hat{I}I}$ for the associated risk profiles $g_{u,a}^{\hat{I}I}, g_{m,a}^{\hat{I}I}, g_{h,a}^{\hat{I}I}, g_{u,b}^{\hat{I}I}, g_{m,b}^{\hat{I}I}$ and $g_{h,b}^{\hat{I}I}$:

$$g_{\cdot,\cdot}^{\hat{I}I} = \omega'\Gamma_{\cdot,\cdot}^{\hat{I}I}\omega + \omega'\Delta^{\hat{I}I}.$$

Setting $\Delta^{\hat{I}I} := 16\mathbb{1}$ offers high and low curvature relative to the slope.

[a] I denotes the identity matrix of corresponding dimension

[b] $\mathbb{1}$ denotes the vector with all components set to 1

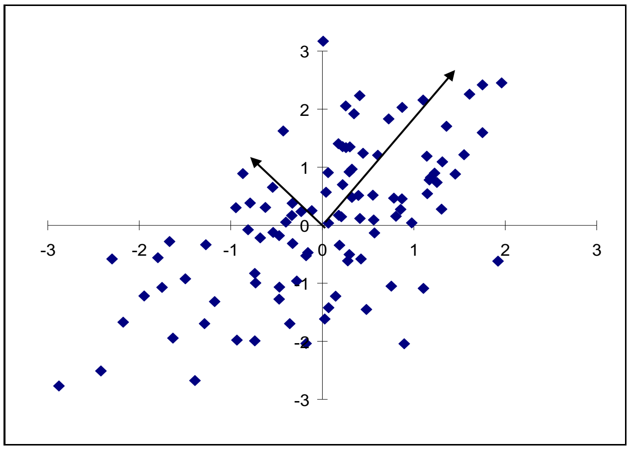

Figure 5: Eigenvectors

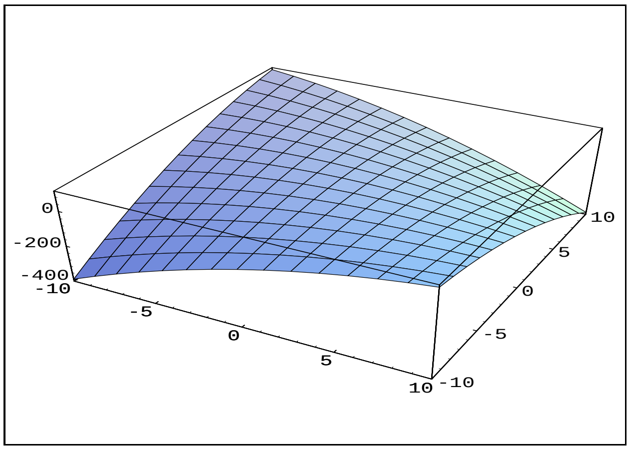

Figure 6: Hessian $\Gamma = V^{-1}DV$

In addition to linear-quadratic risk profiles, UBS [2] has motivated to consider trigonometric profiles, too. We have chosen the 16- and 4-dimensional case:

$$g_{trig}^{II}(\omega) = 100'000 \cdot \sin(\Pi_{i=1}^{16}\omega_i). \tag{10}$$

$$g_{trig}^{\hat{II}}(\omega) = 100'000 \cdot \sin(\Pi_{i=1}^{4}\omega_i). \tag{11}$$

The risk factor $\omega \in \mathbb{R}^{16}$ is multivariate normally distributed with the above correlation matrices $\Sigma_u^{II}, \Sigma_m^{II}$ and Σ_h^{II}.

Environment III)

UBS has motivated the following lattice representation. Applied to a FX-portfolio with K foreign currencies, this results in the evaluation of K risk matrices, where each *risk matrix* (see Table 4) consists of the value change of the underlying portfolio with respect to one pair of risk

G^{III}	exchange rate						
volatility	$p_0 - 3\sigma_p$	$p_0 - 2\sigma_p$	$p_0 - \sigma_p$	p_0	$p_0 + 1\sigma_p$	$p_0 + 2\sigma_p$	$p_0 + 3\sigma_p$
$v_0 - 2\sigma_v$	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{1,6}$	$g_{1,7}$
$v_0 - 1\sigma_v$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$	$g_{2,6}$	$g_{2,7}$
v_0	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$	$g_{3,7}$
$v_0 + 1\sigma_v$	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{4,6}$	$g_{4,7}$
$v_0 + 2\sigma_v$	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$	$g_{5,4}$	$g_{5,5}$	$g_{5,6}$	$g_{5,7}$

Table 4: Lattice representation of risk profile

factors; in case of a FX portfolio the risk factors cross-rate and its volatility are used. Of course, the lattice representation is also applicable to fixed-income or equity portfolios.

The entries $g_{i,j}$ of the matrix G^{III} represent the value change of the underlying portfolio with respect to multiple changes in the price $\pm k\sigma_p$ and in the volatility $\pm k\sigma_v$. 7 cross-rate movements and 5 volatility movements are considered here. The current price and volatility is given by p_0 and v_0. It is noted that by construction $g_{34} = 0$ in the above example. Hence, the value of the portfolio is known for finitely many points. For determining the value change with respect to different factor movements one has to apply adequate inter- or extrapolation, which provides an approximation of the real risk profile g.

Observe that the nonseparability of risk profiles with respect to the prices and with respect to the volatilities is lost when the lattice representation is used in the above way. Only the nonseparability of price and volatility of one underlying currency is taken into account. For measuring that impact we have taken a FX-portfolio with 8 major currencies whose 16-dimensional risk profile has been approximated by a linear-quadratic function. We have received both the FX-portfolio and the quadratic approximation of its risk profile $g^{I\hat{I}I} : \mathbb{R}^{16} \to \mathbb{R}$ by a financial institution.

$$g^{III}(\omega) = \omega'\Gamma^{III}\omega + \omega'\Delta^{III} \tag{12}$$

Unfortunately, the authors have not been authorized to publish the Hessian Γ^{III} and the Δ^{III}-vector. The 7 risk matrices are listed in the Appendix.

Environment IV)

We have received a linear-quadratic risk profile g from a Swiss bank which approximates the value function of an FX-portfolio with 22 foreign currencies and the SFr as reference currency. Considering exchange rates and their volatilities as risk factors, the corresponding space is 46−dimensional. Formally, this risk profile $g^{IV} : \mathbb{R}^{46} \to \mathbb{R}$ is given by

$$g^{IV}(\omega) = \omega'\Gamma^{IV}\omega + \omega'\Delta^{IV'} \tag{13}$$

The logarithm of the price changes and the volatility changes are supposed to be distributed multivariate normal. The associated correlation structure is defined correspondingly with

$$\Sigma^{IV} := \begin{pmatrix} \Sigma_{11}^{IV} & \Sigma_{12}^{IV} \\ \Sigma_{21}^{IV} & \Sigma_{22}^{IV} \end{pmatrix} \in \mathbb{R}^{46\times 46}$$

where Σ_{12}^{IV} incorporates the correlation of exchange rates and volatilities. The variance-covariance matrix Σ^{IV} is available upon request.

Environment V

A FX portfolio PF_1^V has been constructed which covers 8 exchange rates and which consists of 200 different call and put options. The portfolio PF_1^V contains 40% of instruments with maturity 7 days, 40% with maturity 3 months, 10% with maturity 6 months and 10% with maturity one year. 40% of the derivatives are at-the-money, 20% are each mid in-the-money or out-of-the-money and 10% are each deep in- or out-of-the-money.

Subsets of the above portfolio are defined according to the following rules: Portfolio PF_2^V contains in-the-money instruments; portfolio PF_3^V out-of-the-money instruments; portfolio PF_4^V in-the-money calls and out-of-the-money puts; portfolio PF_5^V contains in-the-money puts and out-of-the-money calls. The maturity structures of all these subportfolios are the same.

The variance-covariance matrix Σ^V coincides with the corresponding one in environment II, i.e. $\Sigma^V = \Sigma^{II}$. The Black-Scholes approach is used for valuing the portfolio yielding the corresponding five risk profiles $g_1^V, g_2^V, g_3^V, g_4^V$ and g_5^V.

The instruments and the variance-covariance matrix are listed in the Appendix.

5 Sensitivity of the Value-at-Risk

The aim is to illustrate the sensitivity of the value-at-risk with respect to changes in the market parameter and with respect to various levels. This is done for the above outlined environments independent of the methodology. As mentioned above, the sensitivity of the quantiles with respect to the key parameters, volatility and correlation structure, is one indicator for the model risk, one is exposed to within the value-at-risk approach. This will motivate to pay attention to the slope-curvature relation $\frac{\Gamma_{ii}}{|\Delta_i|}$ along the risk factor components $\omega_i,\ i = 1, \cdots, M$.

In this section we report on the numerical results within the above introduced environments illustrating the sensitivity of value-at-risk with respect to volatility and correlation. This provides information on the model risk.

Environment I

One-dimensional quadratic risk profiles $g^0(\omega), g^+(\omega)$ and $g^-(\omega)$ are considered with ω distributed according to $N(0, \sigma = 1)$ and $N(0, \sigma = 1.3)$. The domain of the risk factor ω has been set to $[-4, 4]$ and may be interpreted as confidence region. The probability that the risk factor is outside that domain is equal to $3 \cdot 10^{-5}$ for $\sigma = 1$ and equal to 10^{-3} for $\sigma = 1.3$. It is recalled that all these profiles have the same maximum loss, set to 4.

Sensitivity of the value-at-risk with respect to the level α: Given that $\omega \sim N(0, 1)$, the probability that ω is greater than 1.64, 1.88, 2.33 respectively, is 0.05, 0.03, 0.01 respectively. Below, these three different levels, $\alpha = 1\%, 3\%$ and 5%, are considered at which the value-at-risk is evaluated for risk profiles g^-, g^0 and g^+. Clearly, the value-at-risk of the convex risk profile g^+ is larger than the value-at-risk of the concave risk profile g^- (see Table 5). This could have been already expected from the Deltas.

α	g^-	g^0	g^+
1%	33.82	58.16	82.49
3%	22.11	47.02	71.93
5%	16.90	41.12	65.30

Table 5: Value-at-risk for different curvatures

Considering the associated relative changes of the value-at-risk with respect to the 5% level, the corresponding numbers are listed in Table 6.

α	g^-	g^0	g^+
1%	200%	141%	126%
3%	131%	114%	110%
5%	100%	100%	100%

Table 6: Changes of the value-at-risk relative to the 5% level

The value-at-risk corresponding to the concave risk profile is more sensitive with respect to the level than the value-at-risk to the convex risk profile. In case of a positive curvature the relative change is less severe. This has not been expected from the Delta, but could have been from the curvature-slope relation $\frac{\Gamma}{|\Delta|}$. It is noted that for the three profiles we have

$$\frac{\Gamma^-}{|\Delta^-|} = -\infty \qquad \frac{\Gamma^0}{|\Delta^0|} = 0 \qquad \frac{\Gamma^+}{|\Delta^+|} = \frac{1}{4}. \tag{14}$$

A Delta-hedged portfolio with negative curvature has a large negative curvature-slope relation which indicates that the value-at-risk number is very sensitive with respect to changes in the level α. The value-at-risk numbers of a portfolio with almost no curvature behave like $1,64 : 1,88 : 2,33$ when levels $\alpha = 1\%, 3\%$ and 5% are chosen. The value-at-risk numbers of a portfolio with positive curvature behave even below the relation $1,64 : 1,88 : 2,33$.

Sensitivity of the value-at-risk with respect to market volatility σ:

Figures 7 and 8 illustrate the value-at-risk curve for $\sigma = 1$ and $\sigma = 1.3$ as a function of level α. Table 7 summarizes the associated numbers for the concave, linear and convex risk profile.

In case the volatility increases by 30%, the value-at-risk changes by 69% for the concave risk profile g^-; obviously, the value-at-risk changes by 30% for the linear risk profile, and it changes by 14%, 18% and respectively 20% for the convex risk profile g^+. These sensitivity results are in line with the curvature-slope relations

$$\frac{\Gamma^-}{|\Delta^-|} = -\infty \qquad \frac{\Gamma^0}{|\Delta^0|} = 0 \qquad \frac{\Gamma^+}{|\Delta^+|} = \frac{1}{4}. \tag{15}$$

The relative changes of the value-at-risk as a function of α based on an increase of the market volatility by 30% is illustrated in Figure 9. Clearly, the constant behaviour in the concave case is caused by the quadratic term of g^-.

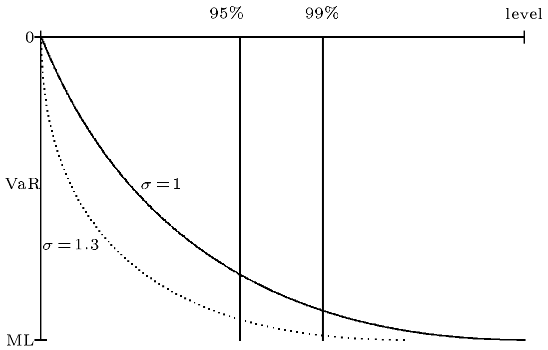

Figure 7: Value-at-risk curve for *positive* curvature

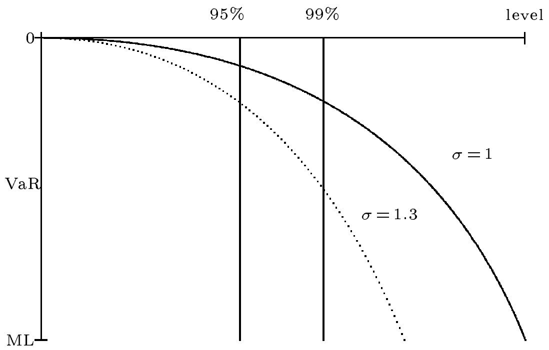

Figure 8: Value-at-risk curve for *negative* curvature

Summarizing, one may state that the value-at-risk is less sensitive with respect to the volatility of the risk factors when the risk profile is convex. This indicates less model risk when the risk exposure of a portfolio is measured. To the contrary, negative curvature in the loss region of the risk profile should be carefully analyzed. This documents that insufficient estimates of the volatility can have significant impact on the identified value-at-risk. In particular, a 30% underestimation of the volatility causes an underestimation of the assessed risk by about 70% in case the risk profile is concave.

Environment II

16− and 4−dimensional linear-quadratic risk profiles of the form

$$g^{II}_{\cdot,\cdot}(\omega) = \omega' \Gamma^{II}_{\cdot,\cdot} \omega + \omega' \Delta^{II} \tag{16}$$

are considered.

The value-at-risk numbers and their sensitivity with respect to market volatility are summarized in the Tables 8 and 9. It is noted that the two curvature-slope relations are

$$\frac{\Gamma_{ii}}{|\Delta_i|} = -\frac{1}{4} \quad \text{for risk profiles } g^{\hat{I}I}_{\cdot,a}, g^{II}_{\cdot,a} \;\; ; \;\; \frac{\Gamma_{ii}}{|\Delta_i|} = -4 \quad \text{for risk profiles } g^{\hat{I}I}_{\cdot,b}, g^{II}_{\cdot,b}. \tag{17}$$

α	g^-			g^0			g^+		
	$\sigma =$ 1.0	$\sigma =$ 1.3	relative change	$\sigma =$ 1.0	$\sigma =$ 1.3	relative change	$\sigma =$ 1.0	$\sigma =$ 1.3	relative change
1%	33.82	57.16	69%	58.16	75.61	30%	82.49	94.04	14%
3%	22.11	37.36	69%	47.02	61.13	30%	71.93	84.88	18%
5%	16.90	28.58	69%	41.12	53.46	30%	65.30	78.33	20%

Table 7: Sensitivity of VaR with respect to market volatility

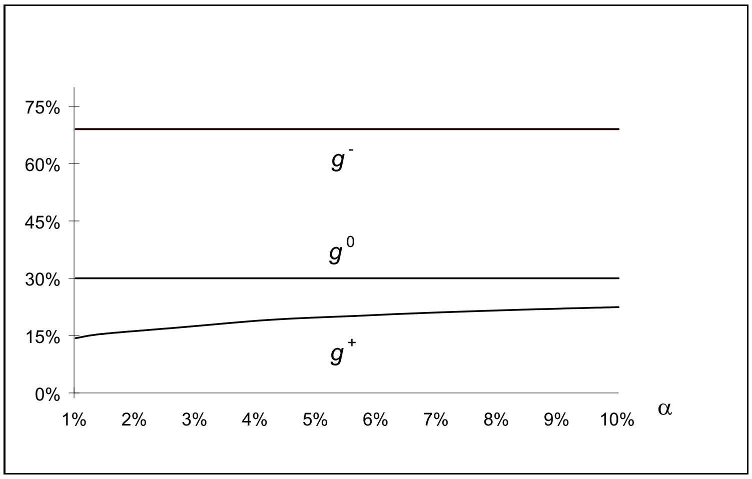

Figure 9: Sensitivity of VaR with respect to market volatility

The correlation structure of the risk factors plays a significant role for the value-at-risk of both curvature-slope relations in the negative definite case. The greater the correlation and the curvature, the higher the value-at-risk. Underestimating the market volatility by 25% yields an underestimation of the value-at-risk by 28% for a risk profile with a small negative curvature-slope relation. Overestimating the volatility by 30% yields an overestimation of the value-at-risk by about 37% for a risk profile with a small negative curvature-slope relation. For risk profiles with larger curvature the value-at-risk is more sensitive with respect to the market volatility. It results an underestimation of the VaR by about 40% and an overestimation of VaR by about 60%. The degree of correlation and the dimension of the risk factor space have less impact on the sensitivity of the value-at-risk with respect to market volatility. It is the curvature of the risk profile which counts for the sensitivity.

Obviously, in case the risk profile is linear the relative changes of the value-at-risk is determined by the relative change of the volatility i.e. the value-at-risk changes by -25% for $\sigma = 0.75$ and by $+30\%$ for $\sigma = 1.3$.

These results confirm that negative curvature and estimating volatility play a significant role for the stability of value-at-risk estimates.

$\alpha = 1\%$	$g^{\hat{I}I}_{\cdot,a}$			$g^{\hat{I}I}_{\cdot,b}$		
	$0.75\Sigma^{\hat{I}I}_{\cdot}$	$\Sigma^{\hat{I}I}_{\cdot}$	$1.3\Sigma^{\hat{I}I}_{\cdot}$	$0.75\Sigma^{\hat{I}I}_{\cdot}$	$\Sigma^{\hat{I}I}_{\cdot}$	$1.30\Sigma^{\hat{I}I}_{\cdot}$
$\Sigma^{\hat{I}I}_{u}$	65.94	92.21	126.61	145.47	241.69	394.14
	-28.48%		37.31%	-39.81 %		63.07 %
$\Sigma^{\hat{I}I}_{m}$	87.09	122.71	168.65	187.99	302.73	475.79
	-29.03%		37.44%	-37.90 %		57.17 %
$\Sigma^{\hat{I}I}_{h}$	123.83	174.45	241.30	271.32	444.32	710.24
	-29.02%		38.32%	-38.94%		59.85%
$\alpha = 3\%$						
$\Sigma^{\hat{I}I}_{u}$	52.98	74.32	101.87	117.79	195.07	314.52
	-28.72%		37.07%	-39.62%		61.23%
$\Sigma^{\hat{I}I}_{m}$	70.17	97.79	133.39	143.23	230.04	366.71
	-28.25%		36.41%	-37.74%		59.41 %
$\Sigma^{\hat{I}I}_{h}$	100.70	140.83	193.57	206.61	334.46	533.70
	-28.50%		37.45%	-38.23%		59.57%
$\alpha = 5\%$						
$\Sigma^{\hat{I}I}_{u}$	46.56	65.43	90.30	102.41	170.92	277.02
	-28.85%		38.01%	-40.08%		62.08%
$\Sigma^{\hat{I}I}_{m}$	60.77	84.67	115.97	122.50	196.55	311.96
	-28.23%		36.96%	-37.68 %		58.71 %
$\Sigma^{\hat{I}I}_{h}$	86.30	120.03	163.88	167.23	271.80	429.27
	-28.11%		36.53%	-38.47 %		57.94 %

Table 8: Sensitivity of the 1%, 3% and 5% VaR for $g^{\hat{I}I}_{\cdot,\cdot}$

$\alpha = 1\%$	$g^{II}_{\cdot,a}$			$g^{II}_{\cdot,b}$		
	$0.75\Sigma^{II}_{\cdot}$	$\Sigma^{II}_{\cdot}$	$1.3\Sigma^{II}_{\cdot}$	$0.75\Sigma^{II}_{\cdot}$	$\Sigma^{II}_{\cdot}$	$1.30\Sigma^{II}_{\cdot}$
Σ^{II}_u	135.45	191.62	265.91	341.66	570.62	922.16
	-29.31%		38.77%	-40.13%		61.60%
Σ^{II}_m	323.75	453.30	623.82	662.31	1'054.15	1'642.66
	-28.58%		37.62%	-37.17%		55.83%
Σ^{II}_h	508.00	716.53	992.62	1'126.59	1'819.94	2'948.40
	-29.10%		38.53%	-38.10%		62.01%
$\alpha = 3\%$						
Σ^{II}_u	112.36	159.41	223.25	292.06	494.00	813.12
	-29.52%		40.05%	-40.88%		64.60%
Σ^{II}_m	257.04	358.72	490.52	502.02	798.22	1'252.36
	-28.35%		36.74%	-37.11%		56.89%
Σ^{II}_h	397.48	555.74	762.66	1'806.97	1'299.65	2'054.97
	-28.48%		37.23%	-39.04%		58.12%
$\alpha = 5\%$						
Σ^{II}_u	100.65	142.98	201.12	270.64	458.12	751.61
	-29.61%		40.66%	-40.92%		64.06%
Σ^{II}_m	223.72	311.81	426.88	433.74	690.27	1'083.10
	-28.25%		36.90%	-37.16 %		56.91 %
Σ^{II}_h	344.76	479.60	654.63	668.88	1'074.59	1'703.00
	-28.12%		36.49%	-37.75 %		58.48 %

Table 9: Sensitivity of the 1%, 3% and 5% VaR for $g^{II}_{\cdot,\cdot}$

In addition to linear-quadratic risk profiles, trigonometric risk profiles have been chosen

$$g^{\hat{I}I}_{trig}(\omega) = 100'000 \cdot \sin(\Pi_{i=1}^{4}\omega_i), \tag{18}$$

$$g^{II}_{trig}(\omega) = 100'000 \cdot \sin(\Pi_{i=1}^{16}\omega_i), \tag{19}$$

to identify the sensitivity of the associated value-at-risk numbers with respect to volatility and correlation. The risk factor ω is multivariate normally distributed with the correlation matrices $\Sigma^{II}_u, \Sigma^{II}_m$ and Σ^{II}_h.

The importance of volatility estimates can be highlighted for trigonometric risk profiles. The maximum loss is 100'000 due the value range $[-1,1]$ of sin. The results in Table 10 illustrate, that an underestimation of the volatility leads to a considerable underestimation of the value-at-risk: in case of high correlation the estimate is about 10 times smaller as the original estimate for the 5% value-at-risk. In case of an overestimation of the volatility the value-at-risk approaches the maximum loss. The results for the 16-dimensional case are even less meaningful.

$\alpha = 1\%$	$g^{\hat{I}I}_{trig}$			g^{II}_{trig}		
	$0.75\Sigma^{\hat{I}I}_{\cdot}$	$\Sigma^{\hat{I}I}_{\cdot}$	$1.3\Sigma^{\hat{I}I}_{\cdot}$	$0.75\Sigma^{II}_{\cdot}$	$\Sigma^{II}_{\cdot}$	$1.3\Sigma^{II}_{\cdot}$
$\Sigma^{\hat{I}I}_u$	80'027	98'570	99'779	131	14'010	95'939
	-18.81%		1.23%	-99.07%		584.79%
$\Sigma^{\hat{I}I}_m$	80'523	98'867	99'771	13'413	86'433	98'615
	-18.55%		0.91%	-84.48%		14.09%
$\Sigma^{\hat{I}I}_h$	94'054	99'028	99'432	97'619	99'246	99'724
	-5.02%		0.41%	-1.64%		0.48%
$\alpha = 3\%$						
$\Sigma^{\hat{I}I}_u$	45'928	86'904	97'623	25	2'617	72'891
	-47.15%		12.33%	-99.03%		4'453.84%
$\Sigma^{\hat{I}I}_m$	46'626	89'905	97'770	392	30'522	86'569
	-48.14%		8.75%	-98.72%		183.63%
$\Sigma^{\hat{I}I}_h$	47'958	89'571	95'503	76'027	92'060	97'116
	-46.46%		6.62%	-17.42%		5.49%
$\alpha = 5\%$						
$\Sigma^{\hat{I}I}_u$	31'218	73'214	93'382	10.15	1'033.09	47'045.35
	-57.36%		27.55%	-99.02%		4'453.84%
$\Sigma^{\hat{I}I}_m$	30'380	75'770	93'145	63.24	7'275.68	67'559.66
	-59.90%		22.93%	-99.13%		828.57%
$\Sigma^{\hat{I}I}_h$	7'173	70'949	87'821	41'162.63	80'926.52	91'826.81
	-89.89%		23.78%	-49.14%		13.47%

Table 10: Sensitivity of the 1%, 3%, 5% VaR for risk profile $g^{\hat{I}I}_{trig}$ and g^{II}_{trig}

Environment III

A 16-dimensional risk profile of a FX-portfolio has been represented by finitely many points through 8 risk matrices (see Appendix) with the components cross-rate and volatility for each of the 8 currencies. These matrices have been used for investigating the impact of separability.

For determining the value change with respect to various factor movements we have applied bilinear and quadratic interpolation, both of which provide an approximation of the real risk profile $g^{III} : \mathbb{R}^{16} \to \mathbb{R}$. The associated value-at-risk numbers are denoted $\hat{v}_{bil}^{+,\alpha}$, $\hat{v}_{bil}^{-,\alpha}$ and $\hat{v}_{qua}^{+,\alpha}$, $\hat{v}_{qua}^{-,\alpha}$ in Tables 11 and 12. As mentioned, the nonseparability of risk profiles with respect to the pairs of risk factors, i.e. with respect to cross-rates and volatilities, is lost. Only the nonseparability of price and volatility of each underlying currency is taken into account.

	g^{III}			$-g^{III}$		
α	$\hat{v}^{\cdot,\alpha}$	$\hat{v}_{bil}^{\cdot,\alpha}$	Error	$\hat{v}^{\cdot,\alpha}$	$\hat{v}_{bil}^{\cdot,\alpha}$	Error
1%	1'142'282	2'562'314	124.32%	2'932'883	989'998	-66.24%
3%	786'830	1'883'783	139.41%	2'042'163	666'072	-67.38%
5%	634'206	1'539'612	142.80%	1'646'894	529'874	-67.80%

Table 11: The impact of separability to the VaR in case of bilinear interpolation

	g^{III}			$-g^{III}$		
α	$\hat{v}^{\cdot,\alpha}$	$\hat{v}_{qua}^{\cdot,\alpha}$	Error	$\hat{v}^{\cdot,\alpha}$	$\hat{v}_{qua}^{\cdot,\alpha}$	Error
1%	1'142'282	2'735'524	139.48%	2'932'883	1'003'032	-65.80%
3%	786'830	1'869'457	137.59%	2'042'163	685'432	-66.44%
5%	634'204	1'539'612	142.76%	1'646'893	529'875	-67.83%

Table 12: The impact of separability to the 5%-VaR in case of quadratic interpolation

These results demonstrate that working with lattice representations may result in a severe over- or underestimation of the value-at-risk. Surprisingly, the way the lattice points are interpolated has less impact. Bilinear and quadratic interpolation yield similiar value-at-risk numbers.

Environment IV

The value-at-risk numbers for the risk profile $g^{IV} : \mathbb{R}^{46} \to \mathbb{R}$ are listed in Table 13. Again, the curvature-slope relations release information on the sensitivity of VaR with respect to the market volatility.

$\alpha = 1\%$	$g^{\hat{IV}}$			g^{IV}		
	$0.75\Sigma^{\hat{IV}}$	$\Sigma^{\hat{IV}}$	$1.3\Sigma^{\hat{IV}}$	$0.75\Sigma^{IV}$	Σ^{IV}	$1.3\Sigma^{IV}$
long	764'911	1'142'282	1'691'640	543'141	799'454	1'173'018
	-33.04%		48.09%	-32.06%		46.73%
short	1'726'241	2'932'883	4'814'333	3'405'054	5'875'258	9'656'543
	-41.14%		64.15%	-42.04%		64.36%
$\alpha = 3\%$						
long	535'230	786'830	1'130'642	353'086	504'993	735'920
	-31.98%		43.70%	-30.08%		45.73%
short	1'219'373	2'042'163	3'342'172	2'342'616	4'120'456	6'878'117
	-40.29%		63.66%	-43.15%		66.93%
$\alpha = 5\%$						
long	440'620	634'206	906'628	266'831	374'758	537'814
	-30.52%		42.95%	-28.80%		43.51%
short	977'594	1'646'894	2'695'266	1'916'609	3'360'034	5'615'786
	-40.64%		63.66%	-42.96%		67.13%

Table 13: Sensitivity of the 1%, 3% and 5% VaR w.r.t. volatility for risk profiles $g^{\hat{IV}}$ and g^{IV}

Environment V

In Table 14 the value-at-risk numbers and their sensitivity are listed for the five FX-portfolio $PF_1^V, PF_2^V, PF_3^V, PF_4^V$ and PF_5^V. The sensitivity of the value-at-risk numbers does not depend on the weights of out-of-the-money, in-the-money and at-the-money options within the portfolio.

$\alpha = 1\%$	long $(+g_\cdot^V)$			short $(-g_\cdot^V)$		
	$0.75\Sigma^V$	Σ^V	$1.3\Sigma^V$	$0.75\Sigma^V$	Σ^V	$1.3\Sigma^V$
PF_1^V	222.08	300.83	401.09	308.58	430.79	588.69
	-26.18%		*33.33%*	*-28.37%*		*36.65%*
PF_2^V	181.12	251.94	345.79	267.06	385.08	541.14
	-28.11%		*37.25%*	*-30.65%*		*40.53%*
PF_3^V	182.89	253.70	347.56	265.29	383.31	539.38
	-27.91%		*37.00%*	*-30.79%*		*40.71%*
PF_4^V	280.31	367.41	458.74	344.83	465.09	621.60
	-23.71%		*24.86%*	*-25.86%*		*33.65%*
PF_5^V	187.72	237.66	314.51	269.67	367.75	505.13
	-21.01%		*32.34%*	*-26.67%*		*37.36%*
$\alpha = 3\%$						
PF_1^V	186.30	253.44	337.55	234.58	331.17	452.22
	-26.49%		*33.19%*	*-29.16%*		*36.55%*
PF_2^V	147.80	209.19	286.71	200.02	293.43	411.65
	-29.35%		*37.06%*	*-31.83%*		*40.29%*
PF_3^V	149.56	210.96	288.48	198.26	291.66	409.88
	-29.10%		*36.75%*	*-32.02%*		*40.53%*
PF_4^V	235.70	308.59	386.84	261.77	366.44	480.06
	-23.62%		*25.36%*	*-28.56%*		*31.01%*
PF_5^V	153.76	196.21	259.80	213.86	279.80	380.02
	-21.63%		*32.41%*	*-23.57%*		*35.82%*
$\alpha = 5\%$						
PF_1^V	166.16	226.60	301.129	198.06	278.59	381.00
	-26.67%		*32.89%*	*-28.91%*		*36.76%*
PF_2^V	131.48	186.08	257.95	169.82	246.58	347.50
	-29.34%		*38.62%*	*-31.13%*		*40.93%*
PF_3^V	133.25	187.85	259.71	168.06	244.82	345.74
	-29.07%		*38.35%*	*-31.35%*		*41.22%*
PF_4^V	208.04	271.83	343.705	222.16	305.13	399.84
	-23.47%		*26.44%*	*-27.19%*		*31.04%*
PF_5^V	133.77	169.61	228.60	186.38	240.09	321.79
	-21.13%		*34.78%*	*-22.37%*		*34.03%*

Table 14: 1%, 3% and 5% VaR of FX-portfolios and its sensitivity w.r.t. market volatility

6 The Barycentric Approximation

The complexity of interaction between time and uncertainty makes practical decision and planning problems to utmost difficult applications of probability and optimization theory. The *Barycentric Approximation* represents a methodology (see Frauendorfer 1992 [11], 1996 [12]) which has been developed for analysing interaction effects between decision making and uncertainty within *stochastic programming* (a field of activity within *mathematical programming*).[c]

Contrary to stochastic control problems, stochastic programs are solved once per period, taking into account periodically updated forecasts of the involved stochastic processes with respect to the future periods. It is today's optimal policy, which is of importance, adopted with respect to the current stochastic dynamics of prices, returns, cash-flows, and also with respect to the optimal policies in future periods, which in turn are adopted with respect to new information on stochastic dynamics. It is this dynamic planning mechanism that characterizes stochastic programming and has received increasing attention in finance in the U.S. and in Great Britain due to the successful and valuable contributions of W.T. Ziemba 1992,1994,1997 [24, 38, 39], M. Dempster 1995 and 1996 [5, 6], J. Mulvey 1994 and 1997 [23, 39] and S. Zenios 1992 and 1995 [36, 38, 37].

The above mentioned dynamic planning mechanism is solved when integration and optimization of value functions has been performed with a prescribed level of accuracy. Barycentric approximation helps overcome the difficulties in the multidimensional integration and optimization of recursively given value functions by sophisticated discretization of the discrete-time stochastic processes (see [12]). In theory, the convergence of the approximate solutions and the corresponding values are enforced by the weak convergence of the discrete measures. In practice, its application within stochastic programming has provided promising results when the decision space is high-dimensional and the probability space is low-dimensional. This has motivated the investigations of Frauendorfer and Königsperger 1995 [13], which represent a first step to applying the barycentric approximation methodology for evaluating profit-and-loss distributions numerically. In [13] the authors have focused on exploiting structural properties of the value functions, like the saddle property, which are valid under reasonable assumptions within stochastic programming but, unfortunately, do not hold when the value functions represent the risk profile of a portfolio with derivatives. Although, convergence of the quantiles is ensured by the weak convergence of the discrete measures, as proclaimed in [13], first numerical results have indicated that the approximation of the quantiles of the associated profit-and-loss distribution are less practical when the portfolio ascertains a reasonable complexity. Even when the level α is kept fixed, the corresponding quantile is approximated with insufficient accuracy by the algorithm introduced in [13]. This reveals that *weak convergence* appears to be not strong enough for evaluating quantiles numerically in a satisfactory way.

The above mentioned experience has motivated the authors to focus on information offered by the barycentric approximation but still unused in the algorithmic procedure [13]. It has been realized that the component which represents the dual to the derived discrete probability measure helps evaluate the quantiles in a better way. Furthermore, the authors learned that working with simplices instead of a product of simplices leads to better results relative to the underlying

[c]For recent textbooks on stochastic programming it is referred to Ermoliev and Wets 1988 [7], Prekopa 1996 [26], Kall and Wallace 1994 [21], Birge and Louveaux 1997 [3]

numerical effort. This gives rise to partition the support of the risk factors into simplices. In this section, the barycentric approximation technique is outlined with the intention to focus on this dual viewpoint for simplicial refinement. For details it is referred to Frauendorfer 1992 [11].

Simplicial truncation of the support

The technique outlined below is based on the property that the outcome of the random vector ω is distributed within a simplex $\hat{\Omega}$. In general, the support of ω may be an arbitrary subset of $\mathbb{R}^M$; in most cases it even covers the entire Euclidean space $\mathbb{R}^M$. For applying the following approach it becomes necessary to approximate the probability space $(\Omega, \mathcal{B}, P)$ by $(\hat{\Omega}, \hat{\mathcal{B}}, \hat{P})$, where $\hat{\Omega}$ represents a simplex, $\hat{\mathcal{B}} := \{\hat{B} | \hat{B} = B \cap \hat{\Omega},\ \forall B \in \mathcal{B}\}$, and $\hat{P}(B) = \frac{P(B \cap \hat{\Omega})}{P(\hat{\Omega})}$.

In order that $(\hat{\Omega}, \hat{\mathcal{B}}, \hat{P})$ approximates $(\Omega, \mathcal{B}, P)$ sufficiently well, it is required $P(\hat{\Omega}) \geq 1 - \epsilon$ for some positive ϵ sufficiently small. Recalling $F_g(\hat{v}) = P\{\omega | g(\omega) \leq \hat{v}\}$ and $\hat{F}_g(\hat{v}) = \hat{P}\{\omega | g(\omega) \leq \hat{v}\}$ yields the following relation

$$\hat{F}_g(\hat{v}) - \epsilon \leq F_g(\hat{v}) \leq \hat{F}_g(\hat{v}) + \epsilon. \tag{20}$$

Hence, any sufficient accurate approximate for $\hat{F}_g$ may be accepted as sufficient accurate for F_g.

For the ease of understanding the simplicial truncation for normally distributed random data is illustrated. The level sets of an uncorrelated normal density are circles. Choose the corresponding radius so that mass $1 - \epsilon$ is lying within that circle and a smallest uniform simplex around that circle (see Figure 10).

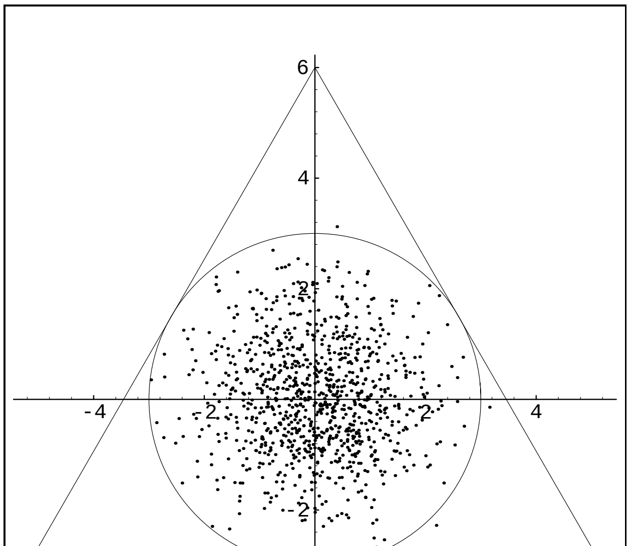

Figure 10: Simplicial coverage in the uncorrelated case

It is well known that the radii of the inner circle and the outer circle - the latter is given by the vertices of the simplex - have a relation of $1 : M$. In case that the random components are correlated we have to determine the Cholesky factorization of the variance-covariance matrix and to apply the associated transformation to the circle and to the uniform simplex. This yields the corresponding ellipsoid surrounded by a regular simplex $\hat{\Omega}$ for which $P(\hat{\Omega}) \geq 1 - \epsilon$ (see Figure 11).

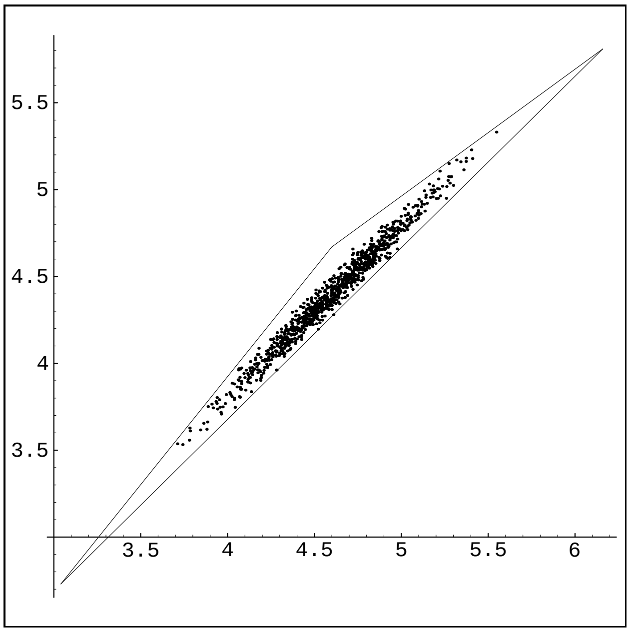

Figure 11: Simplicial coverage in the correlated case

Clearly, the smallest simplex that surrounds the circle is not unique. There are many of these smallest simplices, three of which are illustrated for both the uncorrelated and correlated case in Figure 12. Below, it will be discussed whether the numerical results behave sensitive relative to the choice of the simplex.

So far, it has been the intention to clarify how to realize the simplicial truncation of correlated random components taking into account a prescribed level of accuracy ϵ. It is stressed that the methodology described below poses no assumption on the risk factor distribution P and is applicable for general multivariate or empirical distributions. The technique provides approximations of both, risk profile and truncated probability measure, on the barycenters of simplicial refinements and is called *barycentric approximation.*

Methodology

Given a continuous risk profile $g(\omega) : \hat{\Omega} \subset \mathrm{I\!R}^M \to \mathrm{I\!R}$ and a probability space $(\hat{\Omega}, \hat{\mathcal{B}}, \hat{P})$ with $\hat{\Omega}$ being a simplex, we are interested in approximating the distribution function $\hat{F}_g(\hat{v}) = \hat{P}\{\omega | g(\omega) \leq \hat{v}\}$. This will be achieved by deriving two sequences of piecewise linear approximations $\{L_1^J\}, \{L_2^J\};\ J = 1, 2, \cdots$ that converge pointwise to the risk profile g on $\hat{\Omega}$.

Let Θ denote the set of those probability measures Q on $(\hat{\Omega}, \mathcal{B})$ which coincide in the first moments with those of $\hat{P}$; i.e. Θ consists of those probability measures Q for which

$$\int_{\hat{\Omega}} \omega dQ = \int_{\hat{\Omega}} \omega d\hat{P}, \tag{21}$$

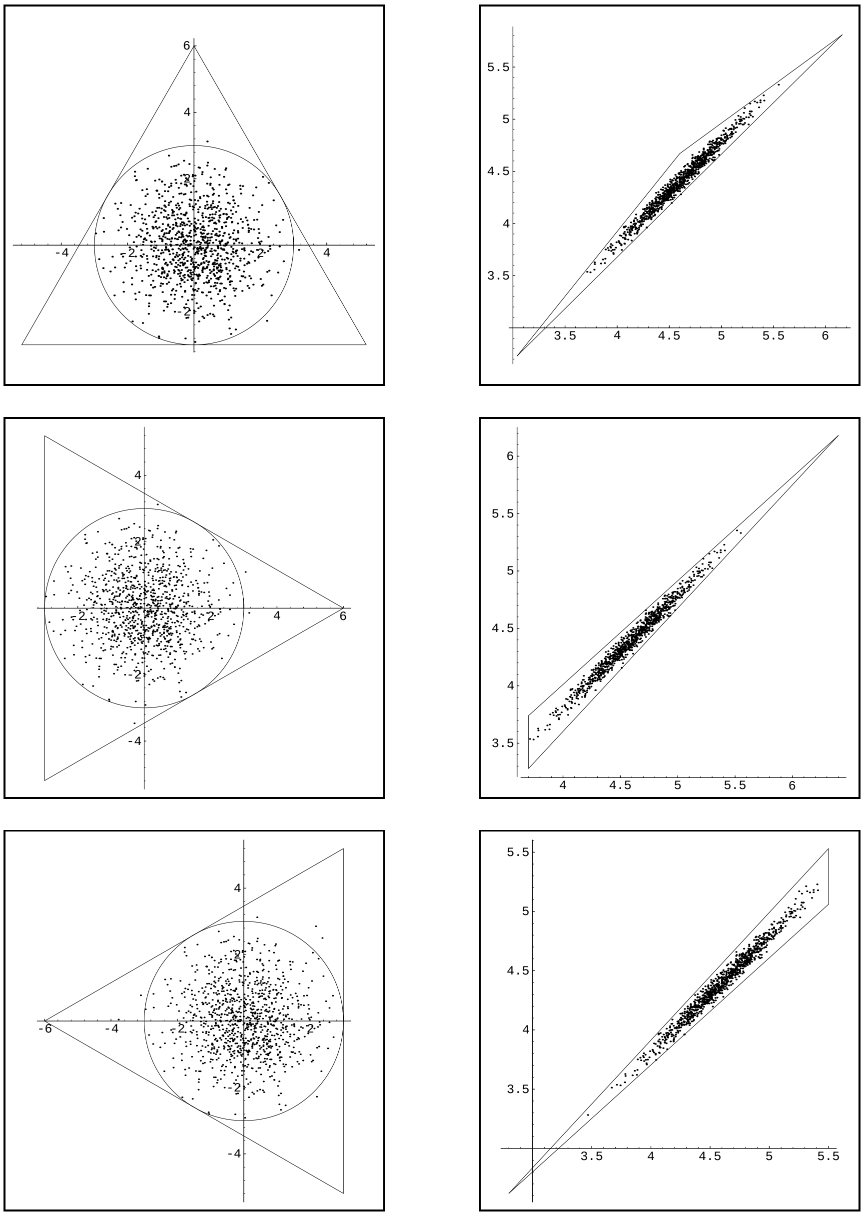

Figure 12: Various illustrations of a simplicial coverage

holds. It is well known (e.g., Stoyan 1983 [31]) that a partial ordering $\leq^{(c)}$ for the set Θ may be defined with respect to the set $\mathcal{C}_{\hat{\Omega}}$ of continuous convex functions relative to the simplex $\hat{\Omega}$:

$$Q_1 \leq^{(c)} Q_2 \Leftrightarrow \int_{\hat{\Omega}} g(\omega) dQ_1 = \int_{\hat{\Omega}} g(\omega) dQ_2, \quad \forall g(\cdot) \in \mathcal{C}_{\hat{\Omega}}. \tag{22}$$

The set of extremal probability measures of Θ taken with respect to $\leq^{(c)}$ is then defined according to

$$\begin{aligned} \inf{}^{(c)}\Theta &:= \{Q_{inn} | Q_{inn} \leq^{(c)} Q, \quad \forall Q, Q_{inn} \in \Theta\}, & (23)\\ \sup{}^{(c)}\Theta &:= \{Q_{out} | Q \leq^{(c)} Q_{out}, \quad \forall Q, Q_{out} \in \Theta\}, & (24) \end{aligned}$$

which represent the solutions of *generalized moment problems* (in the sense of Krein and Nudelman 1977 [22]). It is proven that the sets $\inf^{(c)}\Theta$ and $\sup^{(c)}\Theta$ are singletons; let these solutions be denoted $\hat{Q}_{inn}$ and $\hat{Q}_{out}$ and called *inner* and *outer* discretization. The support of $\hat{Q}_{inn}$ is a singleton, whose element may be viewed as *generalized barycenter* of the simplex $\hat{\Omega}$. Obviously, the barycenter is completely determined by the first moments $\int_{\hat{\Omega}} \omega_m d\hat{P}$ $m = 1, \ldots, M$ which characterize the set Θ. The support of $\hat{Q}_{out}$ is finite and consists of the vertices of the simplex $\hat{\Omega}$. Obviously, the probabilities that are assigned to these vertices are the barycentric weights of the generalized barycenter and, hence, are completely determined by the first moments. For the corresponding formulas of $\hat{Q}_{inn}$ and $\hat{Q}_{out}$ it is referred to [11].

The dual problems to the generalized moment problems (23, 24) are *semiinfinite programs* which are the basis for deriving the desired piecewise linear approximations of the risk profile g.

Let $\mathcal{L}$ denote the set of linear functions $L(\omega)$ for which $L(\cdot) \leq g(\cdot)$ on $\hat{\Omega}$. Similarly, $\mathcal{U}$ denotes the set of linear functions $U(\omega)$ for which $U(\cdot) \geq g(\cdot)$ on $\hat{\Omega}$. Hence, $L(\cdot)$ minorizes $g(\cdot)$ and $U(\cdot)$ majorizes $g(\cdot)$ on $\hat{\Omega}$. Obviously,

$$\begin{aligned} \sup_{L \in \mathcal{L}} \int_{\hat{\Omega}} L(\omega) d\hat{P} &\leq \int_{\hat{\Omega}} g(\omega) d\hat{P}, & (25)\\ \inf_{U \in \mathcal{U}} \int_{\hat{\Omega}} L(\omega) d\hat{P} &\geq \int_{\hat{\Omega}} g(\omega) d\hat{P}. & (26) \end{aligned}$$

The lefthand side in (25, 26) represents the semiinfinite programs which bound the expectation of the risk profile g from below and above. The corresponding solutions are denoted with $\hat{L}$ and $\hat{U}$. $\hat{L}$ is determined by the first-order derivatives of $g(\cdot)$ at the barycenter. $\hat{U}$ is given by the evaluation of the risk profile g at the vertices of $\hat{\Omega}$.

Due to strong duality it is proven for $g(\cdot) \in \mathcal{C}_{\hat{\Omega}}$ that

$$\begin{aligned} \int_{\hat{\Omega}} \hat{L}(\omega) d\hat{P} &= \int_{\hat{\Omega}} g(\omega) d\hat{Q}_{inn}, & (27)\\ \int_{\hat{\Omega}} \hat{U}(\omega) d\hat{P} &= \int_{\hat{\Omega}} g(\omega) d\hat{Q}_{out}. & (28) \end{aligned}$$

Further, as the integral of a linear function is determined by the first moments, it also holds that

$$\begin{aligned} \int_{\hat{\Omega}} \hat{L}(\omega) d\hat{P} &= \int_{\hat{\Omega}} \hat{L}(\omega) d\hat{Q}_{inn}, & (29)\\ \int_{\hat{\Omega}} \hat{U}(\omega) d\hat{P} &= \int_{\hat{\Omega}} \hat{U}(\omega) d\hat{Q}_{out}. & (30) \end{aligned}$$

This way one obtains approximations for the value functions $g(\cdot)$ on $\hat{\Omega}$ as well as for the probability measure $\hat{P}$.

These approximations can be improved with refinements of $\hat{\Omega}$. Let $\mathcal{P}^J$ be a simplicial partition of $\hat{\Omega}$; i.e. $\mathcal{P}^J := \{\hat{\Omega}^j;\ j = 1, \cdots, J\}$ where the subcells are mutually disjoint, i.e. subsimplices whose union equals $\hat{\Omega}$. Applying the above statements to each of these subcells $\hat{\Omega}^j$ $(j = 1, \cdots, J)$ yields improved piecewise linear approximations $\hat{L}^J(\cdot), \hat{U}^J(\cdot)$ and $\hat{Q}^J_{inn}, \hat{Q}^J_{out}$ for the value functions $g(\cdot)$ and for the probability measure $\hat{P}$.

Let $\mathcal{L}^J$ be the set of functions which are piecewise linear with respect to the partition $\mathcal{P}^J$, i.e. the elements of $\mathcal{L}^J$ are linear on the subcells $\hat{\Omega}^j$ of $\mathcal{P}^J$, and which minorize the risk profile g. Analogously, $\mathcal{U}^J$ denotes the set of functions which are piecewise linear with respect to the partition $\mathcal{P}^J$, i.e. the elements of $\mathcal{L}^J$ are linear on the subcells $\hat{\Omega}^j$ of $\mathcal{P}^J$, and which minorize the risk profile g. Then, the solutions to the semiinfinite programs

$$\sup_{L \in \mathcal{L}^{\mathcal{J}}} \int_{\hat{\Omega}} L(\omega) d\hat{P} \quad \leq \quad \int_{\hat{\Omega}} g(\omega) d\hat{P}, \tag{31}$$

$$\inf_{U \in \mathcal{U}^{\mathcal{J}}} \int_{\hat{\Omega}} L(\omega) d\hat{P} \quad \geq \quad \int_{\hat{\Omega}} g(\omega) d\hat{P}. \tag{32}$$

are determined for convex $g(\omega)$ by their first-order derivatives at the barycenters of the cells of $\mathcal{P}^J$. Again, these semiinfinite programs are duals of the corresponding generalized moment problems with unique solutions $\hat{Q}^J_{inn}, \hat{Q}^J_{out}$. In this sense, $\hat{Q}^J_{inn}$, $\hat{Q}^J_{out}$ may be viewed as best discretization of the stochastic risk factors ω relative to the partition $\mathcal{P}^J$. The support of $\hat{Q}^J_{inn}$ and $\hat{Q}^J_{out}$ is finite, whose elements are *generalized barycenters* of the simplices $\hat{\Omega}^j$ $(j = 1, \cdots, J)$. The barycenters are determined by the corresponding conditional first moments. Piecewise linearity of the approximate implies

$$\int_{\hat{\Omega}} \hat{L}^J(\omega) d\hat{P} \quad = \quad \int_{\hat{\Omega}} \hat{L}^J(\omega) d\hat{Q}^J_{inn}, \tag{33}$$

$$\int_{\hat{\Omega}} \hat{U}^J(\omega) d\hat{P} \quad = \quad \int_{\hat{\Omega}} \hat{U}^J(\omega) d\hat{Q}^J_{out}. \tag{34}$$

and, further, due to strong duality in case of convex g

$$\int_{\hat{\Omega}} \hat{L}^J(\omega) d\hat{P} \quad = \quad \int_{\hat{\Omega}} g(\omega) d\hat{Q}^J_{inn}, \tag{35}$$

$$\int_{\hat{\Omega}} \hat{U}^J(\omega) d\hat{P} \quad = \quad \int_{\hat{\Omega}} g(\omega) d\hat{Q}^J_{out}. \tag{36}$$

With the above one may define associate approximate distribution functions with respect to $g(\cdot), \hat{L}^J(\cdot)$ and $\hat{U}^J(\cdot)$ according to

$$\hat{F}^J_{g,inn}(\hat{v}) := \hat{Q}^J_{inn}\{\omega | g(\omega) \leq \hat{v}\}, \tag{37}$$

$$\hat{F}^J_{g,out}(\hat{v}) := \hat{Q}^J_{out}\{\omega | g(\omega) \leq \hat{v}\}, \tag{38}$$

$$\hat{F}^J_L(\hat{v}) := \hat{Q}^J_{inn}\{\omega | L(\omega) \leq \hat{v}\}, \tag{39}$$

$$\hat{F}^J_U(\hat{v}) := \hat{Q}^J_{out}\{\omega | U(\omega) \leq \hat{v}\}. \tag{40}$$

In case the diameters of all subcells tend to 0 for $J \to \infty$ then pointwise convergence of $\hat{L}^J(\cdot), \hat{U}^J(\cdot)$ to $g(\cdot)$ and weak convergence of $\hat{Q}^J_{inn}, \hat{Q}^J_{out}$ to $\hat{P}$ is ensured. This implies pointwise convergence of the approximate distribution functions to $F_g(\hat{v})$ at $\hat{v} \in \mathbb{R}$ at which $F_g(\cdot)$ is continuous. At these $\hat{v}$ it holds

$$\lim_{J\to\infty} \hat{F}^J_{g,inn}(\hat{v}) = \lim_{J\to\infty} \hat{F}^J_{g,out}(\hat{v}) = \lim_{J\to\infty} \hat{F}^J_L(\hat{v}) = \lim_{J\to\infty} \hat{F}^J_U(\hat{v}) = F_g(\hat{v}), \tag{41}$$

Hence, $\hat{F}^J_{g,inn}(\hat{v})$, $\hat{F}^J_{g,out}(\hat{v}), \hat{F}^J_L(\hat{v})$, $\hat{F}^J_U(\hat{v})$ may be accepted as approximation of $F(\hat{v})$. If, additionally, $g(\cdot)$ is a function with monotone first-order derivatives, then the corresponding error can be quantified at each cycle J via the bound. In particular, for convex or concave risk profiles an error bound is given by

$$|\int_{\hat{\Omega}} \hat{U}^J(\omega) d\hat{Q}^J_{out} - \int_{\hat{\Omega}} \hat{L}^J(\omega) d\hat{Q}^J_{inn}|. \tag{42}$$

It is emphasized, that $\hat{F}^J_{g,inn}(\hat{v})$, $\hat{F}^J_{g,out}(\hat{v})$ and, hence, the error bound becomes available by evaluating $g(\cdot)$ at the barycenters. If the sequence of partitions represents successive refinements than the error bound converges monotonously to 0. For nonconvex g the above value may be accepted as a measure for the current inaccuracy, but the convergence to 0 is not necessarily monotonous.

For determining a simplicial partition $\mathcal{P}^{J+1}$ by means of a refinement of $\mathcal{P}^J$, it requires to choose a subcell in $\mathcal{P}^J$ and an adequate edge, subject to which the subcell is split. The experiences achieved with barycentric approximation for solving stochastic programs ([12, 11]) make one aware of the fact that the choice of both subcell and edge is one of the key steps for an appealing speed of convergence. Hence, the numerical effort associated with approximating $F(\cdot) : \mathbb{R} \to [0, 1]$ depends heavily on how sophisticated the sequence of partitions is constructed.

However, the experience has demonstrated that practically the weak convergence of $\hat{Q}^J_{inn}$ and $\hat{Q}^J_{out}$ to $\hat{P}$ is not strong enough for accepting the convergence of the quantiles of $\hat{F}^J_{g,inn}(\hat{v})$ and $\hat{F}^J_{g,out}(\hat{v})$ to the quantiles of $\hat{F}$ as adequate, relative to the Delta-Gamma approximation.

Obviously, taking the piecewise linear approximations L^J and U^J offers more information. In addition to the barycenters and their associated probabilities, which characterize $Q^J_{g,inn}$ and $Q^J_{g,out}$, we obtain with L^J and U^J first-order type approximations of the risk profile g with respect to the subsimplices $\hat{\Omega}^j$. To be more precisely, L^J represents the Delta-approximation of g with respect to the subsimplices, and with U^J a kind of averaged delta approximation on the subsimplices. Due to the fact that the radii of the inner-circle and the outer-circle of a uniform M-dimensional simplex have a relation of $1 : M$, the goodness of U^J decreases with M more severe than the goodness of L^J when the number of refinements are kept fixed. For this reason the authors have concentrated on working with L^J and its associated distribution $\hat{F}^J_L$ as approximations for $+g$ and F. The corresponding VaR-estimate $\hat{v}^{+;\alpha}_{J,L}$ with respect to level α is given by $\hat{F}^J_L(\hat{v}^{+;\alpha}_{J,L}) = \alpha$ and will be benchmarked by the VaR-estimate $\hat{v}^{\alpha}_{\Delta-\Gamma}$ of the Delta-Gamma approximation of g.

Graphical illustration

The support of the risk factor distribution is truncated by a simplex with sufficiently large probability and refined subsequently. In Figure 13 the results of ten refinements are illustrated yielding ten subsimplices with corresponding barycenters, i.e. $J = 10$. The associated piecewise linear approximation L^J is given by the value of g on the barycenters and by the Δ of g at these. The associated discretization Q^J_{inn} is the *inner discretization*. With the information of the delta at each barycenter we have implicitly the information of its sensitivity and hence a second-order information of risk profile $+g$ on $\hat{\Omega}$.

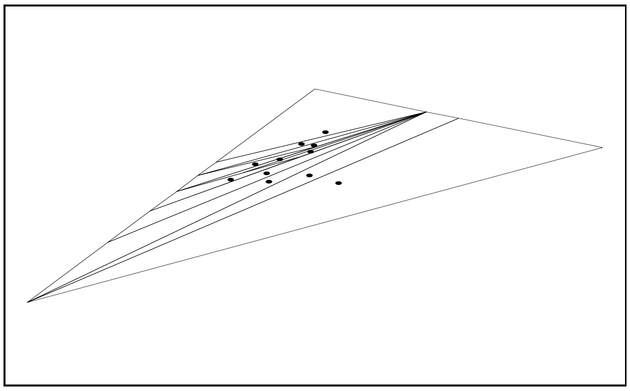

Figure 13: *Inner* discretization

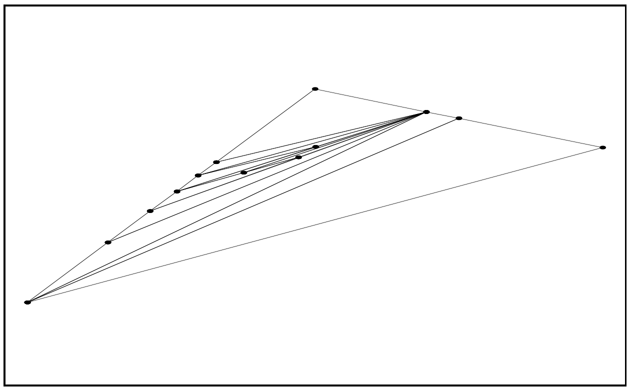

Figure 14: *Outer* discretization

The piecewise linear approximation U^J of g is given by the value of g at the vertices of the subsimplices $\hat{\Omega}^j$ $j = 1, \dots, 10$ and its linear interpolation. The associated discretization Q^J_{out} of the risk factor space is the *outer discretization* whose outcomes are illustrated in Figure 14. The evaluation of the risk profile at the vertices of the multi-dimensional simplex provides *simplicial stress scenarios*, which finally provide additional information on the sensitivity of VaR. However, one has to be aware of the sensitivity of U^J with respect to the choice of the simplex due to the relation $1 : M$ of the radii. Empirically, it has been verified that the piecewise linear approximation L^J is insensitive to different choices of the smallest simplex surrounding the ellipsoid with a prescribed confidence level $1 - \epsilon$.

Surrogate discrete probability measures

As stated above, the methodology ensures convergence of the discrete distributions to the risk factor distribution. The inner and the outer discretizations coincide in the first moments conditioned on the subsimplices with those of the risk factor distribution at any cycle of refinement. The higher moments of the discretized distribution will not coincide with those of the risk factors distribution. This certainly is of practical importance.

Suppose that a random component of the risk factor space is normally distributed with standard deviation σ set to 1, the associated kurtosis is 3. The graphics in Figures 15 and 16 show the evolution of standard deviation σ and the kurtosis of the discretized distributions of that random component.

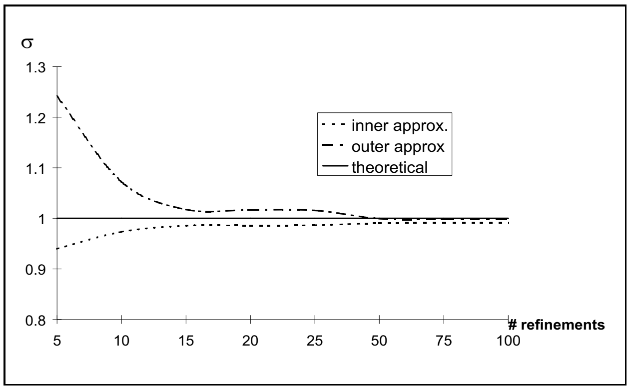

Figure 15: Evolution of the standard deviation subject to refinements

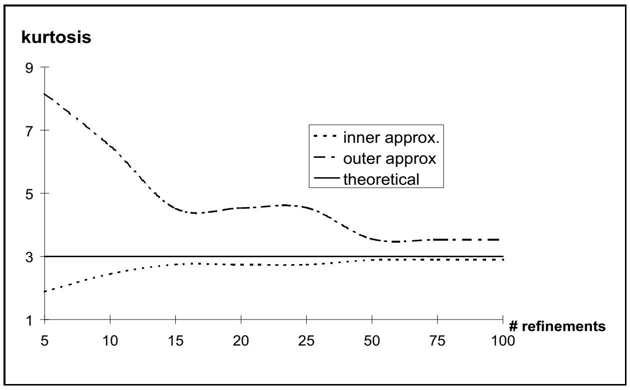

Figure 16: Evolution of the kurtosis subject to refinements

Due to the construction, it happens that the inner approximation Q^J_{inn} likely underestimates σ and kurtosis while the outer approximation Q^J_{out} likely overestimates these moments. Illustrating the inner and outer discretizations (that correspond to various refinement schemes) by these two moments would result in Figure 17. In section 5 the sensitivity of VaR has been evaluated with respect to a 0.75 decrease and an 1.3 increase of the market volatility σ. Below, the goodness of the VaR-approximations obtained by employing the inner discretization should be seen relative to the documented sensitivity of VaR with respect to the various σ. Clearly, with increasing refinements and with the diameters of the subsimplices converging to 0 the associated sequence of standard deviations tends to 1 while the associated sequence of kurtoses tends to 3. However, the critical point is the number of refinements that can be afforded.

Figure 18 illustrates the goodness of the distribution $\hat{F}^J_{g,inn}(\cdot)$ corresponding to the inner dis-

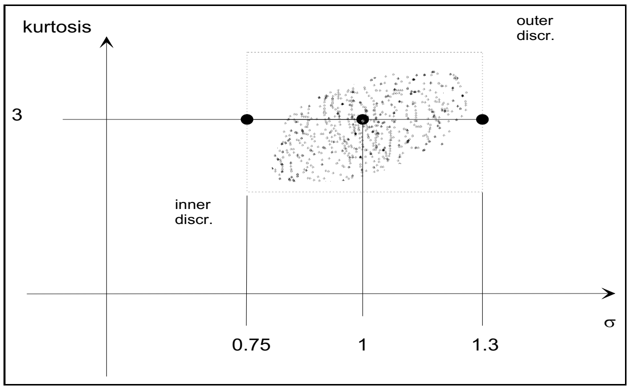

Figure 17: Illustrative representation of the inner and outer discretizations by moments

cretization Q^J_{inn} and the goodness of the distribution $\hat{F}^J_{g,out}(\cdot)$ corresponding to the outer discretization Q^J_{out} relative to the real profit-and-loss distribution $\hat{F}_g$ of some two-dimensional risk profile. Even in the low-dimensional case the outer discretization proves to be not applicable. On the other hand, up to now the authors have not been successful in carrying over the illustrated goodness for the inner approximation with respect to a higher dimensional risk factor space.

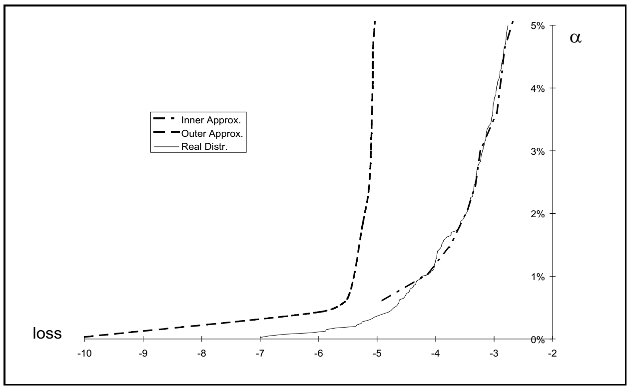

Figure 18: Goodness of the distributions $\hat{F}^J_{g,inn}(\cdot)$ and $\hat{F}^J_{g,out}(\cdot)$

The sensitivity of VaR with respect to over- and underestimation of the market volatility reveals information on the riskiness of the underlying portfolio. One concludes that, nevertheless, a significant difference between the VaR of the outer and the inner approximation is an indicator for high risk exposure. While the inner approximation yields a better discretization of the profit-and-loss distribution, the outer approximation can be considered as proxy for profit-and-loss distributions that correspond to risk factor distributions with " fat tails ". Clearly, for assessing the practical importance of these statements further investigations are required which are postponed to the future.

Convergence of the barycentric approximation

As mentioned, the goodness of the inner and outer discretization depends on the way the refinement schemes are performed. The refinement scheme may be designed with focus on approximating the risk factor distribution or with focus on both the risk profile and the risk factor distribution. Figures 19 and 20 illustrate the convergence beaviour of the 5% VaR corresponding to an one-dimensional risk profile with positive curvature and a normally distributed risk factor.

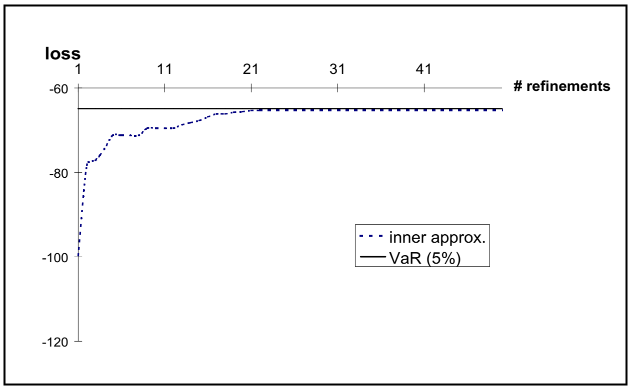

Figure 19: Convergence behavior in case a

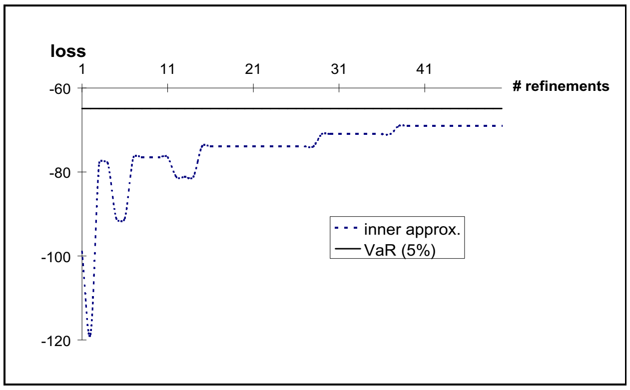

Figure 20: Convergence behavior in case b

Figure 19 illustrates the approximate distribution $\hat{F}^J_{g,inn}(\cdot)$ (curve 1 in Figure 21) associated with a refinement scheme that focus on the riks factor distribution (case a). Figure 20 illustrates the approximate distribution $\hat{F}^J_{g,inn}(\cdot)$ (curve 2 in Figure 21) associated with a refinement scheme that focus on both the risk factor distribution and on the risk profile (case b). Figure 21 reveals the goodness of these approximate distributions (curves 1 and 2) subject to the real distribution F_g (curve 3) relative to a fixed number of refinements for levels between 0.5% and 5%.

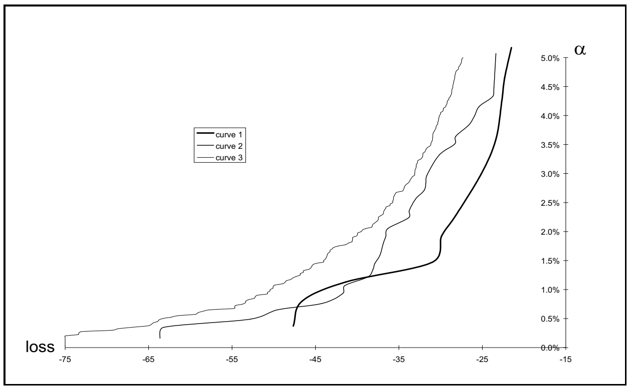

Figure 21: Goodness of real and approximate distributions

Above figures indicate that even in the low-dimensional case it is important to consider the risk factor distribution as well as the risk profile within the refinement scheme.

7 Benchmarking the Barycentric Approximation

In the following, the barycentric approximation is benchmarked by the Delta-Gamma approximation for quadratic and trigonometric risk profiles, for the FX-portfolios and for the ROE warrant on the ABB stock.

Quadratic risk profiles

In environment II, 4− and 16−dimensional linear-quadratic risk profiles are considerd. The corresponding value-at-risk for the long position $v^{-,\alpha}$ and for the short position $v^{-,\alpha}$ are listed with their approximations $v_{B.A.}^{+,\alpha}$ and $v_{B.A.}^{-,\alpha}$ in Tables 15-16 $v_{B.A.}^{+,\alpha}$ and$v_{B.A.}^{-,\alpha}$ are obtained by applying the barycentric approximations after $J = 200$ refinements.

It is observed that the VaR $v^{-,\alpha}$ and $v^{-,\alpha}$ and the accuracy of the barycentric approximation increases with the degree of correlation at each level $\alpha = 1\%, 3\%, 5\%$. Further, the risk profiles $g_{\cdot,a}^{\cdot}$ and $g_{\cdot,b}^{\cdot}$ differ in the curvature-slope relations, i.e. for $g_{\cdot,a}^{\cdot}$ the curvature-slope relation is -0.25, for $g_{\cdot,b}^{\cdot}$ the curvature-slope relation is -4. It is known from section 5 that the sensitivity of the VaR with respect to the market volatility σ increases with decreasing curvature-slope relation. Having in mind the characteristic features of the barycentric approximation it is not surprising that the accuracy of the barycentric approximation decreases with the curvature-slope relation. Also, the accuracy of the barycentric approximation is insensitive with respect to the level α for fixed variance-covariance matrix. Finally, it is stressed that with $v^{+,\alpha}$ and $v^{-,\alpha}$ for levels $\alpha = 1\%, 3\%, 5\%$ the asymmetry of the profit-and-loss distribution is revealed.

$\alpha = 1\%$	$g_a^{\hat{II}}$			$g_b^{\hat{II}}$		
	$v_{B.A.}^{+,\alpha}$	$v^{+,\alpha}$	Error	$v_{B.A.}^{+,\alpha}$	$v^{+,\alpha}$	Error
$\Sigma_u^{\hat{II}}$	85.16	92.21	-7.64%	171.08	241.69	-29.22%
$\Sigma_m^{\hat{II}}$	117.13	122.71	-4.55%	231.63	302.73	-23.49%
$\Sigma_h^{\hat{II}}$	170.80	174.45	-2.09%	373.66	444.32	-15.90%
$\alpha = 3\%$						
$\Sigma_u^{\hat{II}}$	68.80	74.32	-7.43%	136.74	195.07	-29.90%
$\Sigma_m^{\hat{II}}$	93.15	97.79	-4.74%	178.94	230.04	-22.21%
$\Sigma_h^{\hat{II}}$	138.82	140.83	-1.43%	273.59	334.46	-18.20%
$\alpha = 5\%$						
$\Sigma_u^{\hat{II}}$	60.14	65.43	-8.09%	119.97	170.92	-29.81%
$\Sigma_m^{\hat{II}}$	80.53	84.67	-4.89%	152.33	196.55	-22.50%
$\Sigma_h^{\hat{II}}$	117.01	120.03	-2.52%	218.23	271.80	-19.71%

Table 15: Barycentric approximation for risk profiles $g_{\cdot}^{\hat{II}}$

$\alpha = 1\%$	g_a^{II}			g_b^{II}		
	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}$	Error	$v^{+,\alpha}_{B.A.}$	$v^{+,\sigma}$	Error
Σ_u^{II}	156.53	191.62	-18.31%	266.89	570.62	-53.23%
Σ_m^{II}	430.62	453.30	-5.00%	893.22	1'054.15	-15.27%
Σ_h^{II}	708.69	716.53	-1.09%	1'770.61	1'819.94	-2.71%
$\alpha = 3\%$						
Σ_u^{II}	130.76	159.41	-17.97%	222.62	494.00	-54.94%
Σ_m^{II}	337.32	358.72	-5.97%	640.52	798.22	-19.76%
Σ_h^{II}	552.36	555.74	-0.61%	1'262.74	1'299.65	-2.84%
$\alpha = 5\%$						
Σ_u^{II}	116.84	142.98	-18.28%	201.49	458.12	-56.02%
Σ_m^{II}	293.00	311.81	-6.03%	540.08	690.27	-21.76%
Σ_h^{II}	477.03	479.60	-0.54%	1'038.98	1'074.59	-3.31%

Table 16: Barycentric approximation for risk profiles $g^{II}_{\cdot}$

Considering the trigonometric risk profile $g_{trig}^{\hat{II}}$, it is observed that the accuracy of the barycentric approximation is reasonable for level $\alpha = 1\%$ due to the fact that the VaR is bounded by $100'000$. For levels $\alpha = 3\%, 5\%$ the inaccuracy is the largest for the medium correlated case.

$\alpha = 1\%$	$g_{trig}^{\hat{II}}$		
	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}$	Error
$\Sigma_u^{\hat{II}}$	97'519	98'570	-1.08%
$\Sigma_m^{\hat{II}}$	99'798	98'867	0.93%
$\Sigma_h^{\hat{II}}$	97'553	99'028	-1.51%
$\alpha = 3\%$			
$\Sigma_u^{\hat{II}}$	78'904	86'904	-10.14%
$\Sigma_m^{\hat{II}}$	72'340	89'905	-24.28%
$\Sigma_h^{\hat{II}}$	91'094	89'571	1.67%
$\alpha = 5\%$			
$\Sigma_u^{\hat{II}}$	67'630	73'214	-8.26%
$\Sigma_m^{\hat{II}}$	52'669	75'771	-43.86%
$\Sigma_h^{\hat{II}}$	76'710	70'430	8.19%

Table 17: Barycentric approximation for risk profile $g_{trig}^{\hat{II}}$

FX-portfolios

In environment V, five FX portfolio $PF_1^V, PF_2^V, PF_3^V, PF_4^V$ and PF_5^V have been constructed which cover 8 exchange rates and which consist of 200 different call and put options. The Black-Scholes approach is used for valuing the portfolio yielding the corresponding five risk profiles $g_1^V, g_2^V, g_3^V, g_4^V$ and g_5^V.

The corresponding value-at-risk for the long position $v^{-,\alpha}$ and for the short position $v^{-,\alpha}$ are listed with their approximations $v_{B.A.}^{+,\alpha}, v_{B.A.}^{-,\alpha}, v_{\Delta-\Gamma}^{+,\alpha}$ and $v_{\Delta-\Gamma}^{-,\alpha}$ in Tables 18 and 19. $v_{B.A.}^{+,\alpha}$ and $v_{B.A.}^{-,\alpha}$ are obtained by applying the barycentric approximations after $J = 200$ refinements, $v_{\Delta-\Gamma}^{+,\alpha}$ and $v_{\Delta-\Gamma}^{-,\alpha}$ correspond to the linear-quadratic approximation of the risk-profiles.

We recognize that the barycentric and the linear-quadratic approximation yield comparable results (Table 20). The accuracy of both approximations is insensitive with respect to the level α. Again, we realize the asymmetry of the profit-and-loss distribution from $v^{+,\alpha}, v^{-,\alpha}$ or from their approximate values for levels $\alpha = 1\%, 3\%, 5\%$.

$\alpha = 1\%$	long			short		
Underlying	$v_{B.A.}^{+,\alpha}$	$v^{+,\alpha}$	Error	$v_{B.A.}^{-,\alpha}$	$v^{-,\alpha}$	Error
PF_1^V	355.147	300.829	18.06%	374.252	430.789	-13.12%
PF_2^V	299.920	251.936	19.05%	322.736	385.077	-16.19%
PF_3^V	301.755	253.699	18.94%	320.973	383.313	-16.26%
PF_4^V	410.064	367.410	11.61%	415.169	465.090	-10.73%
PF_5^V	276.775	237.659	16.46%	297.099	367.749	-19.21%
$\alpha = 3\%$						
PF_1^V	295.974	253.442	16.78%	285.059	331.165	-13.92%
PF_2^V	249.528	209.193	19.28%	240.130	293.427	-18.16%
PF_3^V	251.292	210.956	19.12%	238.366	291.663	-18.27%
PF_4^V	349.411	308.587	13.23%	313.450	366.440	-14.46%
PF_5^V	233.872	196.208	19.20%	223.441	279.803	-20.14%
$\alpha = 5\%$						
PF_1^V	266.019	226.598	17.40%	240.784	278.591	-13.57%
PF_2^V	222.55	186.084	19.60%	198.591	246.583	-19.46%
PF_3^V	224.313	187.848	19.41%	196.827	244.82	-19.60%
PF_4^V	308.974	271.828	13.67%	266.019	305.127	-12.82%
PF_5^V	210.067	169.613	23.85%	189.731	240.087	-20.97%

Table 18: Barycentric approximation for risk profiles $g_.^V$

$\alpha = 1\%$	long			short		
Underlying	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{+,\alpha}$	Error	$v^{-,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}$	Error
PF_1^V	273.306	300.829	-9.15%	496.877	430.789	15.34%
PF_2^V	225.923	251.936	-10.33%	447.961	385.077	16.33%
PF_3^V	225.923	253.699	-10.95%	447.961	383.313	16.87%
PF_4^V	342.914	367.410	-6.67%	523.187	465.090	12.49%
PF_5^V	206.227	237.659	-13.23%	413.502	367.749	12.44%
$\alpha = 3\%$						
PF_1^V	222.202	253.442	-12.33%	391.006	331.165	18.07%
PF_2^V	180.696	209.193	-13.62%	346.502	293.427	18.09%
PF_3^V	180.696	210.956	-14.34%	346.502	291.663	18.80%
PF_4^V	273.824	308.587	-11.27%	424.043	366.440	15.72%
PF_5^V	164.299	196.208	-16.26%	327.417	279.803	17.02%
$\alpha = 5\%$						
PF_1^V	194.263	226.598	-14.27%	335.929	278.591	20.58%
PF_2^V	156.650	186.084	-15.82%	298.214	246.583	20.94%
PF_3^V	156.650	187.848	-16.61%	298.214	244.820	21.81%
PF_4^V	239.155	271.828	-12.02%	359.393	305.127	17.78%
PF_5^V	141.764	169.613	-16.42%	284.927	240.087	18.68%

Table 19: Delta-Gamma approximation for risk profiles $g^V_\cdot$

Error	$\alpha = 1\%$		$\alpha = 3\%$		$\alpha = 5\%$	
Underlying	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}_{\Delta-\Gamma}$
PF_1^V	18.06%	-9.15%	16.78%	-12.33%	17.40%	-14.27%
PF_2^V	19.05%	-10.33%	19.28%	-13.62%	19.60%	-15.82%
PF_3^V	18.94%	-10.95%	19.12%	14.34%	19.41%	-16.61%
PF_4^V	11.61%	-6.67%	13.23%	-11.27%	13.67%	-12.02%
PF_5^V	16.46%	-13.23%	19.20%	16.26%	23.85%	-16.42%
Underlying	$v^{-,\alpha}_{B.A.}$	$v^{-,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}_{B.A.}$	$v^{-,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}_{B.A.}$	$v^{-,\alpha}_{\Delta-\Gamma}$
PF_1^V	-13.12%	15.34%	-13.92%	18.07%	-13.57%	20.58%
PF_2^V	-16.19%	16.33%	-18.16%	18.09%	-19.46%	20.94%
PF_3^V	-16.26%	16.87%	-18.27%	18.80%	-19.60%	21.81%
PF_4^V	-10.73%	12.49%	-14.46%	15.72%	-12.82%	17.78%
PF_5^V	-19.21%	12.44%	-20.14%	17.02%	-20.97%	18.68%

Table 20: Error of the barycentric and the Delta-Gamma approximations for risk profiles $g^V_\cdot$

ROE warrant

The value-at-risk associated with ROE warrant on the ABB stock is listed with respect to various stock prices in Tables 21 and 22. $v_{B.A.}^{+,\alpha}$ and $v_{B.A.}^{-,\alpha}$ are obtained by applying the barycentric approximations for $J = 100$ refinements, $v_{\Delta-\Gamma}^{+,\alpha}$ and $v_{\Delta-\Gamma}^{-,\alpha}$ correspond to the linear-quadratic approximation of the risk-profiles. The accuracy of the Delta-Gamma approximation is within the range $[-4.52\%, 8.47\%]$ for levels $\alpha = 1\%, 3\%, 5\%$, that of the barycentric approximation is within $[-2.95\%, 3.60\%]$ (see Table 23). Again, it is realized how the asymmetry of the profit-and-loss distribution changes with respect to different prices of the underlying ABB stock.

$\alpha = 1\%$	long			short		
Underlying	$v_{B.A.}^{+,\alpha}$	$v^{+,\alpha}$	Error	$v_{B.A.}^{-,\alpha}$	$v^{-,\alpha}$	Error
1800	402.275	407.056	-1.17%	295.152	291.349	1.31%
1900	397.223	403.917	-1.66%	262.858	257.826	1.95%
2000	388.211	396.749	-2.15%	229.661	223.997	2.53%
2100	374.691	382.426	-2.02%	196.281	190.697	2.93%
2130	370.074	378.123	-2.13%	185.945	180.683	2.91%
2200	360.637	371.608	-2.95%	163.709	158.717	3.15%
$\alpha = 3\%$						
1800	329.088	330.313	-0.37%	255.870	248.882	2.81%
1900	324.292	325.225	-0.29%	231.170	224.033	3.19%
2000	316.261	318.154	-0.59%	205.086	197.950	3.60%
2100	303.502	304.709	-0.40%	176.377	171.595	2.79%
2130	298.672	300.441	-0.59%	168.904	163.559	3.27%
2200	285.573	290.241	-1.61%	149.869	145.687	2.87%
$\alpha = 5\%$						
1800	293.890	293.811	-0.03%	226.890	221.746	2.32%
1900	287.328	288.333	-0.35%	207.383	201.917	2.71%
2000	277.895	279.054	-0.42%	184.834	179.951	2.71%
2100	264.019	265.605	-0.60%	161.336	158.470	1.81%
2130	259.508	261.284	-0.68%	154.558	151.669	1.90%
2200	249.626	250.410	-0.31%	138.152	136.166	1.46%

Table 21: Barycentric approximation for the ROE warrant

On the whole, it is recognized that the barycentric is competitive with the Delta-Gamma approximation. The accuracy of both approximations is insensitive with respect to the level α. The asymmetry of the profit-and-loss distribution is realized from the value-at-risk proxies at various levels. Having in mind that the barycentric approximation is applicable for general multivariate distributions, for which the variance-covariance matrix exists, makes this methodology a promising tool for developing and improving risk assessment systems for both trading and management.

$\alpha = 1\%$	long			short		
Underlying	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{+,\alpha}$	Error	$v^{-,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}$	Error
1800	418.560	407.056	2.83%	302.531	291.349	3.84%
1900	409.234	403.917	1.32%	272.693	257.826	5.77%
2000	389.240	396.749	-1.89%	239.327	223.997	6.84%
2100	377.615	382.426	-1.26%	205.527	190.697	7.78%
2130	369.296	378.123	-2.33%	195.977	180.683	8.46%
2200	354.800	371.608	-4.52%	172.167	158.717	8.47%
$\alpha = 3\%$						
1800	334.473	330.313	1.26%	247.144	248.882	-0.70%
1900	325.085	325.225	-0.04%	224.860	224.033	0.37%
2000	312.928	318.154	-1.64%	199.744	197.950	0.91%
2100	296.848	304.709	-2.58%	174.399	171.595	1.63%
2130	293.088	300.441	-2.45%	166.341	163.559	1.70%
2200	283.806	290.241	-2.22%	149.365	145.687	2.52%
$\alpha = 5\%$						
1800	295.990	293.811	0.74%	217.352	221.746	-1.98%
1900	285.413	288.333	-1.01%	198.393	201.917	-1.75%
2000	271.974	279.054	-2.54%	177.008	179.951	-1.64%
2100	258.168	265.605	-2.80%	156.616	158.470	-1.17%
2130	253.719	261.284	-2.90%	150.037	151.669	-1.08%
2200	241.574	250.410	-3.53%	134.090	136.166	-1.52%

Table 22: Delta-Gamma approximation for the ROE warrant

Error	$\alpha = 1\%$		$\alpha = 3\%$		$\alpha = 5\%$	
Underlying	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}_{\Delta-\Gamma}$	$v^{+,\alpha}_{B.A.}$	$v^{+,\alpha}_{\Delta-\Gamma}$
1800	-1.17%	2.83%	-0.37%	1.26%	-0.03%	0.74%
1900	-1.66%	1.32%	-0.29%	-0.04%	-0.35%	-1.01%
2000	-2.15%	-1.89%	-0.59%	-1.64%	-0.42%	-2.54%
2100	-2.02%	-1.26%	-0.40%	-2.58%	-0.60%	-2.80%
2130	-2.13%	-2.33%	-0.59%	-2.45%	-0.68%	-2.90%
2200	-2.95%	-4.52%	-1.61%	-2.22%	-0.31%	-3.53%
Underlying	$v^{-,\alpha}_{B.A.}$	$v^{-,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}_{B.A.}$	$v^{-,\alpha}_{\Delta-\Gamma}$	$v^{-,\alpha}_{B.A.}$	$v^{-,\alpha}_{\Delta-\Gamma}$
1800	1.31%	3.84%	2.81%	-0.70%	2.32%	-1.98%
1900	1.95%	5.77%	3.19%	0.37%	2.71%	-1.75%
2000	2.53%	6.84%	3.60%	0.91%	2.71%	-1.64%
2100	2.93%	7.78%	2.79%	1.63%	1.81%	-1.17%
2130	2.91%	8.46%	3.27%	1.70%	1.90%	-1.08%
2200	3.15%	8.47%	2.87%	2.52%	1.46%	-1.52%

Table 23: Error of the barycentric and the Delta-Gamma approximations for the ROE warrant

8 Conclusions and Outlook

We have started with assessing the sensitivity of the value-at-risk with respect to market volatility. The curvature-slope relation of a risk profile evaluated at the current market situation reveals information on that sensitivity, and, hence, on the model risk, one is exposed to when the parameter volatility is over- or underestimated.

Based on the experience that weak convergence of the probability measures is not strong enough to receive adequate approximates of the value-at-risk with a reasonable numerical effort, we have focused on the dual view of the barycentric approximation. It is the piecewise linearization of the risk profile over a simplicial partition of the risk factor space, which provides an appealing approximation of the profit-and-loss distribution. The generalized barycenters of the subsimplices may be viewed as distinguished market scenarios subject to which the portfolio has to be evaluated. The approach is also applicable to nonnormally distributed risk factors for which the variance-covariance matrix exists. This matrix helps design the simplicial truncation of the support so that a mass $1-\epsilon$ is considered with ϵ being sufficiently small with respect to the level α. The refinement scheme is of importance for achieving a convergence behaviour of the approximate risk-return patterns which is of practical usage. As learned from the low-dimensional case already, both risk factor space and risk profile have to be taken into account to fulfill the needs for applicability in the high-dimensional case. The accuracy of the barycentric approximation correlates with the sensitivity of the VaR due to its characteristic features and has proven to be competitive with the Delta-Gamma approximation.

In the above, the barycentric approximation has been applied to a ROE warrant on the ABB stock and has been benchmarked by the Delta-Gamma approximation for various risk profiles. The numerical results have illustrated the risk-return pattern of the ROE warrant and the various risk profiles. The asymmetry of the risk-return pattern reveals the risk attitude of the investors which proclaim those risk profiles.

This work is seen as one step towards controling and managing market risk with the value-at-risk approach. The key for being efficient lies in an adequate mapping of the risk-return pattern that corresponds to the underlying portfolio. Based on the current developments and the achieved experiences the focus of future research activities is therefore posed on various issues. The way the refinement process of the simplicial partition is designed is still judged as rather crude by the authors. The information on the variability of the slope and the curvature, which is available locally at the barycenters and the vertices of the subsimplices, reveals the goodness of the piecewise linearization. This information is still unused although it appears to be of major importance to the authors, not only for assessing the model risk but also for improving the convergence behaviour of the approximate risk-return pattern.

As mentioned, the barycenters represent market scenarios subject to which the portfolio has to be analyzed. These portfolio values and their sensitivities provide the basis for evaluating and implementing optimized hedging activities. This requires that the value-at-risk approach becomes embedded into a stochastic optimization problem. The challenge of these future activities lies in determining the dynamic of its risk-return pattern and how this can be incorporated in an active portfolio management.

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Appendix

Environment I)

The one-dimensional risk profiles are considered

$$g_0(\omega) = -25\omega\,, \qquad g^-(\omega) = \frac{-25}{4}\omega^2\,2 \qquad g^+(\omega) = \frac{25}{4}\omega^2 - 50\omega$$

with $\omega \sim N(0, \sigma = 1)$ and $N(0, \sigma = 1.3)$

Environment II)

4-dimensional linear-quadratic risk profiles of the form

$$g^{\hat{I}I}_{\cdot,\cdot}(\omega) = \omega\Gamma^{\hat{I}I}_{\cdot,\cdot}\omega + \omega'\Delta^{\hat{I}I}$$

are considered with $\Delta^{\hat{I}I} := 16 \cdot \mathbb{1}$ ($\mathbb{1} \in \mathrm{I\!R}^4$). The Hessian matrices $\Gamma^{\hat{I}I}_{u,a}, \Gamma^{\hat{I}I}_{m,a}, \Gamma^{\hat{I}I}_{h,a}, \Gamma^{\hat{I}I}_{u,b}, \Gamma^{\hat{I}I}_{m,b}$ and $\Gamma^{\hat{I}I}_{h,b}$ are given through $\Gamma^{\hat{I}I}_{\cdot,\cdot} = [V^{\hat{I}I}_{\cdot}]^{-1} D^{\hat{I}I}_{\cdot} V^{\hat{I}I}_{\cdot}$ where $D^{\hat{I}I}_{\cdot}$ represents $D^{\hat{I}I}_{a} := -4I, D^{\hat{I}I}_{b} := -16I$, ($I \in \mathrm{I\!R}^{4\times 4}$) and $V^{\hat{I}I}_{\cdot,\cdot}$ represent the matrix of eigenvectors $V^{\hat{I}I}_{u}, V^{\hat{I}I}_{m}, V^{\hat{I}I}_{h}$ corresponding to the variance-covariance matrices $\Sigma^{\hat{I}I}_{u}, \Sigma^{\hat{I}I}_{m}, \Sigma^{\hat{I}I}_{h}$.

$$\Sigma^{\hat{I}I}_{u} := I \qquad \Sigma^{\hat{I}I}_{m} := \begin{pmatrix} 1.000 & 0.049 & 0.387 & 0.074 \\ 0.049 & 1.000 & 0.477 & 0.309 \\ 0.387 & 0.477 & 1.000 & 0.227 \\ 0.074 & 0.309 & 0.227 & 1.000 \end{pmatrix} \qquad \Sigma^{\hat{I}I}_{h} := \begin{pmatrix} 1.000 & 0.755 & 0.855 & 0.811 \\ 0.755 & 1.000 & 0.884 & 0.957 \\ 0.855 & 0.884 & 1.000 & 0.911 \\ 0.811 & 0.957 & 0.911 & 1.000 \end{pmatrix}$$

16-dimensional linear-quadratic risk profiles of the form

$$g^{II}_{\cdot,\cdot}(\omega) = \omega\Gamma^{II}_{\cdot,\cdot}\omega + \omega'\Delta^{II}$$

are considered with $\Delta^{II} := 16 \cdot \mathbb{1}$ ($\mathbb{1} \in \mathrm{I\!R}^{16}$). The Hessian matrices $\Gamma^{II}_{u,a}, \Gamma^{II}_{m,a}, \Gamma^{II}_{h,a}, \Gamma^{II}_{u,b}, \Gamma^{II}_{m,b}$ and $\Gamma^{II}_{h,b}$ are given through $\Gamma^{II}_{\cdot,\cdot} = [V^{II}_{\cdot}]^{-1} D^{II}_{\cdot} V^{II}_{\cdot}$ where $D^{II}_{\cdot}$ represents $D^{II}_{a} := -4I, D^{II}_{b} := -16I$, ($I \in \mathrm{I\!R}^{4\times 4}$) and $V^{II}_{\cdot,\cdot}$ represent the matrix of eigenvectors $V^{II}_{u}, V^{II}_{m}, V^{II}_{h}$ corresponding to the variance-covariance matrices $\Sigma^{II}_{u}, \Sigma^{II}_{m}, \Sigma^{II}_{h}$.

$$\Sigma_m^{II} := \left(\Sigma_m^{II}(1) \vdots \Sigma_m^{II}(2)\right)$$

$$\Sigma_m^{II}(1) := \begin{pmatrix}
1.000 & 0.190 & 0.230 & 0.410 & 0.210 & 0.440 & 0.190 & 0.450 \\
0.190 & 1.000 & 0.360 & 0.430 & 0.380 & 0.400 & 0.240 & 0.280 \\
0.230 & 0.360 & 1.000 & 0.480 & 0.360 & 0.300 & 0.360 & 0.410 \\
0.410 & 0.430 & 0.480 & 1.000 & 0.440 & 0.380 & 0.180 & 0.380 \\
0.210 & 0.380 & 0.360 & 0.440 & 1.000 & 0.430 & 0.410 & 0.310 \\
0.440 & 0.400 & 0.300 & 0.380 & 0.430 & 1.000 & 0.160 & 0.370 \\
0.190 & 0.240 & 0.360 & 0.180 & 0.410 & 0.160 & 1.000 & 0.440 \\
0.450 & 0.280 & 0.410 & 0.380 & 0.310 & 0.370 & 0.440 & 1.000 \\
0.210 & 0.170 & 0.330 & 0.450 & 0.400 & 0.280 & 0.190 & 0.200 \\
0.400 & 0.380 & 0.310 & 0.350 & 0.260 & 0.430 & 0.410 & 0.460 \\
0.510 & 0.250 & 0.310 & 0.290 & 0.210 & 0.180 & 0.400 & 0.300 \\
0.420 & 0.160 & 0.490 & 0.210 & 0.270 & 0.190 & 0.440 & 0.300 \\
0.300 & 0.500 & 0.280 & 0.360 & 0.440 & 0.320 & 0.330 & 0.180 \\
0.240 & 0.530 & 0.300 & 0.300 & 0.380 & 0.290 & 0.340 & 0.210 \\
0.460 & 0.420 & 0.510 & 0.310 & 0.260 & 0.440 & 0.420 & 0.190 \\
0.360 & 0.520 & 0.410 & 0.410 & 0.160 & 0.540 & 0.320 & 0.290
\end{pmatrix}$$

$$\Sigma_u^{II}(2) := \begin{pmatrix}
0.210 & 0.400 & 0.510 & 0.420 & 0.300 & 0.240 & 0.460 & 0.360 \\
0.170 & 0.380 & 0.250 & 0.160 & 0.500 & 0.530 & 0.420 & 0.520 \\
0.330 & 0.310 & 0.310 & 0.490 & 0.280 & 0.300 & 0.510 & 0.410 \\
0.450 & 0.350 & 0.290 & 0.210 & 0.360 & 0.300 & 0.310 & 0.410 \\
0.400 & 0.260 & 0.210 & 0.270 & 0.440 & 0.380 & 0.260 & 0.160 \\
0.280 & 0.430 & 0.180 & 0.190 & 0.320 & 0.290 & 0.440 & 0.540 \\
0.190 & 0.410 & 0.400 & 0.440 & 0.330 & 0.340 & 0.420 & 0.320 \\
0.200 & 0.460 & 0.300 & 0.300 & 0.180 & 0.210 & 0.190 & 0.290 \\
1.000 & 0.390 & 0.240 & 0.200 & 0.210 & 0.160 & 0.260 & 0.160 \\
0.390 & 1.000 & 0.350 & 0.460 & 0.280 & 0.460 & 0.370 & 0.470 \\
0.240 & 0.350 & 1.000 & 0.340 & 0.290 & 0.400 & 0.310 & 0.210 \\
0.200 & 0.460 & 0.340 & 1.000 & 0.310 & 0.420 & 0.410 & 0.510 \\
0.210 & 0.280 & 0.290 & 0.310 & 1.000 & 0.310 & 0.300 & 0.200 \\
0.160 & 0.460 & 0.400 & 0.420 & 0.310 & 1.000 & 0.400 & 0.500 \\
0.260 & 0.370 & 0.310 & 0.410 & 0.300 & 0.400 & 1.000 & 0.260 \\
0.160 & 0.470 & 0.210 & 0.510 & 0.200 & 0.500 & 0.260 & 1.000
\end{pmatrix}$$

$$\Sigma_h^{II} := \left(\Sigma_h^{II}(1) \vdots \Sigma_h^{II}(2) \right)$$

$$\Sigma_h^{II}(1) := \begin{pmatrix} 1.000 & 0.900 & 0.900 & 0.900 & 0.910 & 0.900 & 0.900 & 0.900 \\ 0.900 & 1.000 & 0.900 & 0.910 & 0.930 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 1.000 & 0.900 & 0.970 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.910 & 0.900 & 1.000 & 0.950 & 0.900 & 0.900 & 0.900 \\ 0.910 & 0.930 & 0.970 & 0.950 & 1.000 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 1.000 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 1.000 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 1.000 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \end{pmatrix}$$

$$\Sigma_h^{II}(2) := \begin{pmatrix} 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 1.000 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 1.000 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 1.000 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 1.000 & 0.900 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 1.000 & 0.900 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 1.000 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 1.000 & 0.900 \\ 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 0.900 & 1.000 \end{pmatrix}$$

Environment III)

The risk profile of a FX-portfolio which consists of 7 foreign currencies and one home-curreny is given by a linear-quadratic approximation of the form

$$g^{III}(\omega) = \omega'\Gamma^{III}\omega + \omega'\Delta^{III}$$

The authors have not been authorized to publish the Hessian Γ^{III} and the sensitivity vector Δ^{III}. The 8×8 variance-covariance matrix $\Sigma^{III} := \left(\Sigma^{III}(1) \vdots \Sigma^{III}(2)\right)$ is given below. The risk profile g^{III} is evaluated at lattice points. The corresponding values are given in 7 risk matrices.

G_1^{III}	exchange rate						
vola	0.58729393	0.68429077	0.78128761	0.87828444	0.97528128	1.07227812	1.16927496
0.640	242346.879	124717.674	58269.413	43002.095	78915.721	166010.290	304285.803
0.840	220845.831	103216.627	36768.365	21501.048	57414.673	144509.242	282784.755
1.102	199344.783	81715.579	15267.318	0.000	35913.626	123008.195	261283.707
1.446	177843.736	60214.531	-6233.730	-21501.048	14412.578	101507.147	239782.660
1.898	156342.688	38713.483	-27734.778	-43002.095	- 7088.470	80006.099	218281.612

G_2^{III}	exchange rate						
vola	0.72484985	0.75524300	0.78563615	0.81602930	0.84642245	0.87681560	0.90720875
0.945	-293650.71	-143538.75	-46284.95	-1889.33	-10351.88	-71672.60	-185851.50
0.987	-292706.05	-142594.08	-45340.29	-944.66	-9407.21	-70727.94	-184906.83
1.031	-291761.39	-141649.42	-44395.62	0.00	-8462.55	-69783.28	-183962.17
1.077	-290816.72	-140704.76	-43450.96	944.66	-7517.89	-68838.61	-183017.51
1.125	-289872.06	-139760.09	-42506.30	1889.33	-6573.22	-67893.95	-182072.84

G_3^{III}	exchange rate						
vola	0.21275973	0.22207817	0.23139660	0.24071504	0.25003347	0.25935191	0.26867034
0.903	155612.00	128330.74	90762.02	42905.83	-15237.81	-83668.92	-162387.50
0.955	134159.08	106877.82	69309.10	21452.92	-36690.73	-105121.84	-183840.42
1.009	112706.16	85424.90	47856.18	0.00	-58143.65	-126574.76	-205293.33
1.067	91253.24	63971.99	26403.27	-21452.92	-79596.56	-148027.68	-226746.25
1.128	69800.33	42519.07	4950.35	-42905.83	-101049.48	-169480.59	-248199.17

G_4^{III}	exchange rate						
vola	1.34460898	1.52286058	1.70111218	1.87936379	2.05761539	2.23586700	2.41411860
0.801	3372872.6	1360580.3	187728.8	-145681.9	360348.2	1705819.1	3890730.9
0.978	3445713.5	1433421.2	260569.8	-72840.9	433189.2	1778660.1	3963571.8
1.195	3518554.5	1506262.2	333410.7	0.0	506030.1	1851501.0	4036412.8
1.460	3591395.4	1579103.1	406251.6	72840.9	578871.1	1924342.0	4109253.7
1.783	3664236.3	1651944.0	479092.6	145681.9	651712.0	1997182.9	4182094.6

G_5^{III}	exchange rate						
vola	0.00062862	0.00068458	0.00074055	0.00079652	0.00085248	0.00090845	0.00096442
0.878	44237.090	44264.447	44291.719	44318.906	44346.007	44373.023	44399.954
0.937	22077.637	22104.994	22132.266	22159.453	22186.554	22213.570	22240.501
1.000	-81.816	-54.459	-27.187	0.000	27.101	54.118	81.049
1.067	-22241.269	-22213.911	-22186.640	-22159.453	-22132.351	-22105.335	-22078.404
1.139	-44400.721	-44373.364	-44346.092	-44318.906	-44291.804	-44264.788	-44237.857

G_6^{III}	exchange rate						
vola	0.00849952	0.00938254	0.01026557	0.01114859	0.01203161	0.01291463	0.01379765
0.819	-1161.595	-1173.152	-1184.617	-1195.989	-1207.269	-1218.457	-1229.552
0.906	-563.600	-575.158	-586.622	-597.995	-609.275	-620.462	-631.557
1.001	34.394	22.837	11.372	0.000	-11.280	-22.467	-33.562
1.106	632.389	620.832	609.367	597.995	586.715	575.527	564.432
1.223	1230.384	1218.826	1207.362	1195.989	1184.709	1173.522	1162.427

G_7^{III}	exchange rate						
vola	0.83574806	0.95933177	1.08291549	1.20649920	1.33008292	1.45366664	1.57725035
0.670	-4412041.9	-1584570.1	-67.7	341465.4	-559970.8	-2704376.3	-6091751.1
0.871	-4582774.6	-1755302.8	-170800.4	170732.7	-730703.5	-2875109.0	-6262483.9
1.132	-4753507.2	-1926035.5	-341533.1	0.0	-901436.2	-3045841.7	-6433216.6
1.470	-4924239.9	-2096768.2	-512265.8	-170732.7	-1072168.9	-3216574.4	-6603949.2
1.910	-5094972.6	-2267500.9	-682998.5	-341465.4	-1242901.6	-3387307.1	-6774681.9

$$\Sigma^{III}(1) :=$$

$$\begin{pmatrix}
9.40839E-3 & 0.0 & 2.07260E-3 & 7.30798E-4 & 1.48116E-2 & 4.25605E-6 & 6.93523E-5 & 1.15292E-2 \\
0.00000E+0 & 1.0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 \\
2.07260E-3 & 0.0 & 9.23744E-4 & 2.71820E-4 & 4.05768E-3 & 1.10062E-6 & 1.43007E-5 & 2.78224E-3 \\
7.30798E-4 & 0.0 & 2.71820E-4 & 8.68332E-5 & 1.35651E-3 & 3.74751E-7 & 4.98385E-6 & 9.67013E-4 \\
1.48116E-2 & 0.0 & 4.05768E-3 & 1.35651E-3 & 3.17736E-2 & 7.87192E-6 & 1.00724E-4 & 1.95008E-2 \\
4.25605E-6 & 0.0 & 1.10062E-6 & 3.74751E-7 & 7.87192E-6 & 3.13222E-9 & 3.19452E-8 & 5.07775E-6 \\
6.93523E-5 & 0.0 & 1.43007E-5 & 4.98385E-6 & 1.00724E-4 & 3.19452E-8 & 7.79727E-7 & 8.63521E-5 \\
1.15292E-2 & 0.0 & 2.78224E-3 & 9.67013E-4 & 1.95008E-2 & 5.07775E-6 & 8.63521E-5 & 1.52729E-2 \\
2.17830E-2 & 0.0 & 4.61752E-3 & 1.70295E-3 & 3.50482E-2 & 7.57144E-6 & 1.61194E-4 & 2.83638E-2 \\
0.00000E+0 & 0.0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 \\
2.98084E-3 & 0.0 & 7.54805E-4 & 2.54977E-4 & 5.04428E-3 & 1.06366E-6 & 2.13606E-5 & 3.98052E-3 \\
3.93784E-3 & 0.0 & 8.99363E-4 & 3.19347E-4 & 6.48908E-3 & 1.33384E-6 & 2.87406E-5 & 5.22457E-3 \\
1.58546E-2 & 0.0 & 3.50816E-3 & 1.27116E-3 & 2.77425E-2 & 6.17867E-6 & 1.11982E-4 & 2.06606E-2 \\
4.45107E-3 & 0.0 & 1.15017E-3 & 3.78895E-4 & 9.14281E-3 & 2.38666E-6 & 3.11205E-5 & 5.89115E-3 \\
7.74265E-3 & 0.0 & 1.45165E-3 & 5.58066E-4 & 1.23132E-2 & 2.62607E-6 & 5.62159E-5 & 1.00909E-2 \\
2.10292E-2 & 0.0 & 4.49132E-3 & 1.65468E-3 & 3.37994E-2 & 7.38097E-6 & 1.53227E-4 & 2.72949E-2
\end{pmatrix}$$

$$\Sigma^{III}(2) :=$$

$$\begin{pmatrix}
2.17830E-2 & 0.0 & 2.98084E-3 & 3.93784E-3 & 1.58546E-2 & 4.45107E-3 & 7.74265E-3 & 2.10292E-2 \\
0.00000E+0 & 0.0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 \\
4.61752E-3 & 0.0 & 7.54805E-4 & 8.99363E-4 & 3.50816E-3 & 1.15017E-3 & 1.45165E-3 & 4.49132E-3 \\
1.70295E-3 & 0.0 & 2.54977E-4 & 3.19347E-4 & 1.27116E-3 & 3.78895E-4 & 5.58066E-4 & 1.65468E-3 \\
3.50482E-2 & 0.0 & 5.04428E-3 & 6.48908E-3 & 2.77425E-2 & 9.14281E-3 & 1.23132E-2 & 3.37994E-2 \\
7.57144E-6 & 0.0 & 1.06366E-6 & 1.33384E-6 & 6.17867E-6 & 2.38666E-6 & 2.62607E-6 & 7.38097E-6 \\
1.61194E-4 & 0.0 & 2.13606E-5 & 2.87406E-5 & 1.11982E-4 & 3.11205E-5 & 5.62159E-5 & 1.53227E-4 \\
2.83638E-2 & 0.0 & 3.98052E-3 & 5.22457E-3 & 2.06606E-2 & 5.89115E-3 & 1.00909E-2 & 2.72949E-2 \\
7.39196E-2 & 0.0 & 9.77800E-3 & 1.38763E-2 & 5.28698E-2 & 1.36384E-2 & 2.55393E-2 & 7.09405E-2 \\
0.00000E+0 & 1.0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 & 0.00000E+0 \\
9.77800E-3 & 0.0 & 1.89784E-3 & 2.31367E-3 & 7.22954E-3 & 1.94745E-3 & 3.21934E-3 & 9.43676E-3 \\
1.38763E-2 & 0.0 & 2.31367E-3 & 3.06933E-3 & 1.01138E-2 & 2.62059E-3 & 4.64699E-3 & 1.33759E-2 \\
5.28698E-2 & 0.0 & 7.22954E-3 & 1.01138E-2 & 4.00026E-2 & 1.08520E-2 & 1.82480E-2 & 5.08983E-2 \\
1.36384E-2 & 0.0 & 1.94745E-3 & 2.62059E-3 & 1.08520E-2 & 4.24756E-3 & 4.54948E-3 & 1.32111E-2 \\
2.55393E-2 & 0.0 & 3.21934E-3 & 4.64699E-3 & 1.82480E-2 & 4.54948E-3 & 1.00026E-2 & 2.45396E-2 \\
7.09405E-2 & 0.0 & 9.43676E-3 & 1.33759E-2 & 5.08983E-2 & 1.32111E-2 & 2.45396E-2 & 6.85050E-2
\end{pmatrix}$$

Environment IV)

The risk profile of a FX-portfolio which consists of 22 foreign currencies is given by a linear-quadratic approximation of the form

$$g^{IV}(\omega) = \omega'\Gamma^{IV}\omega + \omega'\Delta^{IV}$$

The authors have not been authorized to publish the Hessian Γ^{IV} and the sensitivity vector Δ^{IV}. The 46×46 variance-covariance matrix Σ^{IV} is available upon request.

Environment V)

Below, the various instruments of the five FX-porfolios $PF_1^V, PF_2^V, PF_3^V, PF_4^V, PF_5^V$ are listed. 8 Currencies are considered with currency 2 representing the home currency. The 8 variance-covariance matrix Σ^V is taken from environment III, i.e. $\Sigma^V := \Sigma^{III}$.

Environment V)

Portfolio : PF1 **Currency : 1**

Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	0.87828444	0.87301474	10%	11.043898%	one week
400	Call	0.87828444	0.87564959	10%	11.043898%	one week
800	Call	0.87828444	0.87828444	10%	11.043898%	one week
400	Call	0.87828444	0.88091930	10%	11.043898%	one week
200	Call	0.87828444	0.88355415	10%	11.043898%	one week
200	Call	0.87828444	0.81241311	10%	11.043898%	3 months
400	Call	0.87828444	0.84534878	10%	11.043898%	3 months
800	Call	0.87828444	0.87828444	10%	11.043898%	3 months
400	Call	0.87828444	0.91122011	10%	11.043898%	3 months
200	Call	0.87828444	0.94415578	10%	11.043898%	3 months
50	Call	0.87828444	0.74654178	10%	11.043898%	6 months
100	Call	0.87828444	0.81241311	10%	11.043898%	6 months
200	Call	0.87828444	0.87828444	10%	11.043898%	6 months
100	Call	0.87828444	0.94415578	10%	11.043898%	6 months
50	Call	0.87828444	1.01002711	10%	11.043898%	6 months
50	Call	0.87828444	0.61479911	10%	11.043898%	one year
100	Call	0.87828444	0.74654178	10%	11.043898%	one year
200	Call	0.87828444	0.87828444	10%	11.043898%	one year
100	Call	0.87828444	1.01002711	10%	11.043898%	one year
50	Call	0.87828444	1.14176978	10%	11.043898%	one year
200	Put	0.87828444	0.87301474	10%	11.043898%	one week
400	Put	0.87828444	0.87564959	10%	11.043898%	one week
800	Put	0.87828444	0.87828444	10%	11.043898%	one week
400	Put	0.87828444	0.88091930	10%	11.043898%	one week
200	Put	0.87828444	0.88355415	10%	11.043898%	one week
200	Put	0.87828444	0.81241311	10%	11.043898%	3 months
400	Put	0.87828444	0.84534878	10%	11.043898%	3 months
800	Put	0.87828444	0.87828444	10%	11.043898%	3 months
400	Put	0.87828444	0.91122011	10%	11.043898%	3 months
200	Put	0.87828444	0.94415578	10%	11.043898%	3 months
50	Put	0.87828444	0.74654178	10%	11.043898%	6 months
100	Put	0.87828444	0.81241311	10%	11.043898%	6 months
200	Put	0.87828444	0.87828444	10%	11.043898%	6 months
100	Put	0.87828444	0.94415578	10%	11.043898%	6 months
50	Put	0.87828444	1.01002711	10%	11.043898%	6 months
50	Put	0.87828444	0.61479911	10%	11.043898%	one year
100	Put	0.87828444	0.74654178	10%	11.043898%	one year
200	Put	0.87828444	0.87828444	10%	11.043898%	one year
100	Put	0.87828444	1.01002711	10%	11.043898%	one year
50	Put	0.87828444	1.14176978	10%	11.043898%	one year

Portfolio :	**PF2**		**Home**	**currency:**	No FX	Contracts

Portfolio :	**PF1**			**Currency :**	**3**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	0.81602930	0.81113312	10%	3.724517%	one week
400	Call	0.81602930	0.81358121	10%	3.724517%	one week
800	Call	0.81602930	0.81602930	10%	3.724517%	one week
400	Call	0.81602930	0.81847738	10%	3.724517%	one week
200	Call	0.81602930	0.82092547	10%	3.724517%	one week
200	Call	0.81602930	0.75482710	10%	3.724517%	3 months
400	Call	0.81602930	0.78542820	10%	3.724517%	3 months
800	Call	0.81602930	0.81602930	10%	3.724517%	3 months
400	Call	0.81602930	0.84663039	10%	3.724517%	3 months
200	Call	0.81602930	0.87723149	10%	3.724517%	3 months
50	Call	0.81602930	0.69362490	10%	3.724517%	6 months
100	Call	0.81602930	0.75482710	10%	3.724517%	6 months
200	Call	0.81602930	0.81602930	10%	3.724517%	6 months
100	Call	0.81602930	0.87723149	10%	3.724517%	6 months
50	Call	0.81602930	0.93843369	10%	3.724517%	6 months
50	Call	0.81602930	0.57122051	10%	3.724517%	one year
100	Call	0.81602930	0.69362490	10%	3.724517%	one year
200	Call	0.81602930	0.81602930	10%	3.724517%	one year
100	Call	0.81602930	0.93843369	10%	3.724517%	one year
50	Call	0.81602930	1.06083808	10%	3.724517%	one year
200	Put	0.81602930	0.81113312	10%	3.724517%	one week
400	Put	0.81602930	0.81358121	10%	3.724517%	one week
800	Put	0.81602930	0.81602930	10%	3.724517%	one week
400	Put	0.81602930	0.81847738	10%	3.724517%	one week
200	Put	0.81602930	0.82092547	10%	3.724517%	one week
200	Put	0.81602930	0.75482710	10%	3.724517%	3 months
400	Put	0.81602930	0.78542820	10%	3.724517%	3 months
800	Put	0.81602930	0.81602930	10%	3.724517%	3 months
400	Put	0.81602930	0.84663039	10%	3.724517%	3 months
200	Put	0.81602930	0.87723149	10%	3.724517%	3 months
50	Put	0.81602930	0.69362490	10%	3.724517%	6 months
100	Put	0.81602930	0.75482710	10%	3.724517%	6 months
200	Put	0.81602930	0.81602930	10%	3.724517%	6 months
100	Put	0.81602930	0.87723149	10%	3.724517%	6 months
50	Put	0.81602930	0.93843369	10%	3.724517%	6 months
50	Put	0.81602930	0.57122051	10%	3.724517%	one year
100	Put	0.81602930	0.69362490	10%	3.724517%	one year
200	Put	0.81602930	0.81602930	10%	3.724517%	one year
100	Put	0.81602930	0.93843369	10%	3.724517%	one year
50	Put	0.81602930	1.06083808	10%	3.724517%	one year

Portfolio : PF1 **Currency : 4**

Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	0.24071504	0.23927075	10%	3.871148%	one week
400	Call	0.24071504	0.23999289	10%	3.871148%	one week
800	Call	0.24071504	0.24071504	10%	3.871148%	one week
400	Call	0.24071504	0.24143718	10%	3.871148%	one week
200	Call	0.24071504	0.24215933	10%	3.871148%	one week
200	Call	0.24071504	0.22266141	10%	3.871148%	3 months
400	Call	0.24071504	0.23168822	10%	3.871148%	3 months
800	Call	0.24071504	0.24071504	10%	3.871148%	3 months
400	Call	0.24071504	0.24974185	10%	3.871148%	3 months
200	Call	0.24071504	0.25876867	10%	3.871148%	3 months
50	Call	0.24071504	0.20460778	10%	3.871148%	6 months
100	Call	0.24071504	0.22266141	10%	3.871148%	6 months
200	Call	0.24071504	0.24071504	10%	3.871148%	6 months
100	Call	0.24071504	0.25876867	10%	3.871148%	6 months
50	Call	0.24071504	0.27682229	10%	3.871148%	6 months
50	Call	0.24071504	0.16850053	10%	3.871148%	one year
100	Call	0.24071504	0.20460778	10%	3.871148%	one year
200	Call	0.24071504	0.24071504	10%	3.871148%	one year
100	Call	0.24071504	0.27682229	10%	3.871148%	one year
50	Call	0.24071504	0.31292955	10%	3.871148%	one year
200	Put	0.24071504	0.23927075	10%	3.871148%	one week
400	Put	0.24071504	0.23999289	10%	3.871148%	one week
800	Put	0.24071504	0.24071504	10%	3.871148%	one week
400	Put	0.24071504	0.24143718	10%	3.871148%	one week
200	Put	0.24071504	0.24215933	10%	3.871148%	one week
200	Put	0.24071504	0.22266141	10%	3.871148%	3 months
400	Put	0.24071504	0.23168822	10%	3.871148%	3 months
800	Put	0.24071504	0.24071504	10%	3.871148%	3 months
400	Put	0.24071504	0.24974185	10%	3.871148%	3 months
200	Put	0.24071504	0.25876867	10%	3.871148%	3 months
50	Put	0.24071504	0.20460778	10%	3.871148%	6 months
100	Put	0.24071504	0.22266141	10%	3.871148%	6 months
200	Put	0.24071504	0.24071504	10%	3.871148%	6 months
100	Put	0.24071504	0.25876867	10%	3.871148%	6 months
50	Put	0.24071504	0.27682229	10%	3.871148%	6 months
50	Put	0.24071504	0.16850053	10%	3.871148%	one year
100	Put	0.24071504	0.20460778	10%	3.871148%	one year
200	Put	0.24071504	0.24071504	10%	3.871148%	one year
100	Put	0.24071504	0.27682229	10%	3.871148%	one year
50	Put	0.24071504	0.31292955	10%	3.871148%	one year

Portfolio : PF1 **Currency : 5**

Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	1.87936379	1.86808761	10%	9.484678%	one week
400	Call	1.87936379	1.87372570	10%	9.484678%	one week
800	Call	1.87936379	1.87936379	10%	9.484678%	one week
400	Call	1.87936379	1.88500188	10%	9.484678%	one week
200	Call	1.87936379	1.89063997	10%	9.484678%	one week
200	Call	1.87936379	1.73841150	10%	9.484678%	3 months
400	Call	1.87936379	1.80888765	10%	9.484678%	3 months
800	Call	1.87936379	1.87936379	10%	9.484678%	3 months
400	Call	1.87936379	1.94983993	10%	9.484678%	3 months
200	Call	1.87936379	2.02031607	10%	9.484678%	3 months
50	Call	1.87936379	1.59745922	10%	9.484678%	6 months
100	Call	1.87936379	1.73841150	10%	9.484678%	6 months
200	Call	1.87936379	1.87936379	10%	9.484678%	6 months
100	Call	1.87936379	2.02031607	10%	9.484678%	6 months
50	Call	1.87936379	2.16126836	10%	9.484678%	6 months
50	Call	1.87936379	1.31555465	10%	9.484678%	one year
100	Call	1.87936379	1.59745922	10%	9.484678%	one year
200	Call	1.87936379	1.87936379	10%	9.484678%	one year
100	Call	1.87936379	2.16126836	10%	9.484678%	one year
50	Call	1.87936379	2.44317293	10%	9.484678%	one year
200	Put	1.87936379	1.86808761	10%	9.484678%	one week
400	Put	1.87936379	1.87372570	10%	9.484678%	one week
800	Put	1.87936379	1.87936379	10%	9.484678%	one week
400	Put	1.87936379	1.88500188	10%	9.484678%	one week
200	Put	1.87936379	1.89063997	10%	9.484678%	one week
200	Put	1.87936379	1.73841150	10%	9.484678%	3 months
400	Put	1.87936379	1.80888765	10%	9.484678%	3 months
800	Put	1.87936379	1.87936379	10%	9.484678%	3 months
400	Put	1.87936379	1.94983993	10%	9.484678%	3 months
200	Put	1.87936379	2.02031607	10%	9.484678%	3 months
50	Put	1.87936379	1.59745922	10%	9.484678%	6 months
100	Put	1.87936379	1.73841150	10%	9.484678%	6 months
200	Put	1.87936379	1.87936379	10%	9.484678%	6 months
100	Put	1.87936379	2.02031607	10%	9.484678%	6 months
50	Put	1.87936379	2.16126836	10%	9.484678%	6 months
50	Put	1.87936379	1.31555465	10%	9.484678%	one year
100	Put	1.87936379	1.59745922	10%	9.484678%	one year
200	Put	1.87936379	1.87936379	10%	9.484678%	one year
100	Put	1.87936379	2.16126836	10%	9.484678%	one year
50	Put	1.87936379	2.44317293	10%	9.484678%	one year

Portfolio :	**PF1**			**Currency :**	**6**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	0.00079652	0.00079174	10%	7.026373%	one week
400	Call	0.00079652	0.00079413	10%	7.026373%	one week
800	Call	0.00079652	0.00079652	10%	7.026373%	one week
400	Call	0.00079652	0.00079891	10%	7.026373%	one week
200	Call	0.00079652	0.00080130	10%	7.026373%	one week
200	Call	0.00079652	0.00073678	10%	7.026373%	3 months
400	Call	0.00079652	0.00076665	10%	7.026373%	3 months
800	Call	0.00079652	0.00079652	10%	7.026373%	3 months
400	Call	0.00079652	0.00082639	10%	7.026373%	3 months
200	Call	0.00079652	0.00085626	10%	7.026373%	3 months
50	Call	0.00079652	0.00067704	10%	7.026373%	6 months
100	Call	0.00079652	0.00073678	10%	7.026373%	6 months
200	Call	0.00079652	0.00079652	10%	7.026373%	6 months
100	Call	0.00079652	0.00085626	10%	7.026373%	6 months
50	Call	0.00079652	0.00091599	10%	7.026373%	6 months
50	Call	0.00079652	0.00055756	10%	7.026373%	one year
100	Call	0.00079652	0.00067704	10%	7.026373%	one year
200	Call	0.00079652	0.00079652	10%	7.026373%	one year
100	Call	0.00079652	0.00091599	10%	7.026373%	one year
50	Call	0.00079652	0.00103547	10%	7.026373%	one year
200	Put	0.00079652	0.00079174	10%	7.026373%	one week
400	Put	0.00079652	0.00079413	10%	7.026373%	one week
800	Put	0.00079652	0.00079652	10%	7.026373%	one week
400	Put	0.00079652	0.00079891	10%	7.026373%	one week
200	Put	0.00079652	0.00080130	10%	7.026373%	one week
200	Put	0.00079652	0.00073678	10%	7.026373%	3 months
400	Put	0.00079652	0.00076665	10%	7.026373%	3 months
800	Put	0.00079652	0.00079652	10%	7.026373%	3 months
400	Put	0.00079652	0.00082639	10%	7.026373%	3 months
200	Put	0.00079652	0.00085626	10%	7.026373%	3 months
50	Put	0.00079652	0.00067704	10%	7.026373%	6 months
100	Put	0.00079652	0.00073678	10%	7.026373%	6 months
200	Put	0.00079652	0.00079652	10%	7.026373%	6 months
100	Put	0.00079652	0.00085626	10%	7.026373%	6 months
50	Put	0.00079652	0.00091599	10%	7.026373%	6 months
50	Put	0.00079652	0.00055756	10%	7.026373%	one year
100	Put	0.00079652	0.00067704	10%	7.026373%	one year
200	Put	0.00079652	0.00079652	10%	7.026373%	one year
100	Put	0.00079652	0.00091599	10%	7.026373%	one year
50	Put	0.00079652	0.00103547	10%	7.026373%	one year

Portfolio : PF1 **Currency : 7**

Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	0.01114859	0.01108170	10%	7.920480%	one week
400	Call	0.01114859	0.01111514	10%	7.920480%	one week
800	Call	0.01114859	0.01114859	10%	7.920480%	one week
400	Call	0.01114859	0.01118203	10%	7.920480%	one week
200	Call	0.01114859	0.01121548	10%	7.920480%	one week
200	Call	0.01114859	0.01031244	10%	7.920480%	3 months
400	Call	0.01114859	0.01073052	10%	7.920480%	3 months
800	Call	0.01114859	0.01114859	10%	7.920480%	3 months
400	Call	0.01114859	0.01156666	10%	7.920480%	3 months
200	Call	0.01114859	0.01198473	10%	7.920480%	3 months
50	Call	0.01114859	0.00947630	10%	7.920480%	6 months
100	Call	0.01114859	0.01031244	10%	7.920480%	6 months
200	Call	0.01114859	0.01114859	10%	7.920480%	6 months
100	Call	0.01114859	0.01198473	10%	7.920480%	6 months
50	Call	0.01114859	0.01282088	10%	7.920480%	6 months
50	Call	0.01114859	0.00780401	10%	7.920480%	one year
100	Call	0.01114859	0.00947630	10%	7.920480%	one year
200	Call	0.01114859	0.01114859	10%	7.920480%	one year
100	Call	0.01114859	0.01282088	10%	7.920480%	one year
50	Call	0.01114859	0.01449316	10%	7.920480%	one year
200	Put	0.01114859	0.01108170	10%	7.920480%	one week
400	Put	0.01114859	0.01111514	10%	7.920480%	one week
800	Put	0.01114859	0.01114859	10%	7.920480%	one week
400	Put	0.01114859	0.01118203	10%	7.920480%	one week
200	Put	0.01114859	0.01121548	10%	7.920480%	one week
200	Put	0.01114859	0.01031244	10%	7.920480%	3 months
400	Put	0.01114859	0.01073052	10%	7.920480%	3 months
800	Put	0.01114859	0.01114859	10%	7.920480%	3 months
400	Put	0.01114859	0.01156666	10%	7.920480%	3 months
200	Put	0.01114859	0.01198473	10%	7.920480%	3 months
50	Put	0.01114859	0.00947630	10%	7.920480%	6 months
100	Put	0.01114859	0.01031244	10%	7.920480%	6 months
200	Put	0.01114859	0.01114859	10%	7.920480%	6 months
100	Put	0.01114859	0.01198473	10%	7.920480%	6 months
50	Put	0.01114859	0.01282088	10%	7.920480%	6 months
50	Put	0.01114859	0.00780401	10%	7.920480%	one year
100	Put	0.01114859	0.00947630	10%	7.920480%	one year
200	Put	0.01114859	0.01114859	10%	7.920480%	one year
100	Put	0.01114859	0.01282088	10%	7.920480%	one year
50	Put	0.01114859	0.01449316	10%	7.920480%	one year

Portfolio :	**PF1**			**Currency :**	**8**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
200	Call	1.20649920	1.19926021	10%	10.243166%	one week
400	Call	1.20649920	1.20287971	10%	10.243166%	one week
800	Call	1.20649920	1.20649920	10%	10.243166%	one week
400	Call	1.20649920	1.21011870	10%	10.243166%	one week
200	Call	1.20649920	1.21373820	10%	10.243166%	one week
200	Call	1.20649920	1.11601176	10%	10.243166%	3 months
400	Call	1.20649920	1.16125548	10%	10.243166%	3 months
800	Call	1.20649920	1.20649920	10%	10.243166%	3 months
400	Call	1.20649920	1.25174292	10%	10.243166%	3 months
200	Call	1.20649920	1.29698664	10%	10.243166%	3 months
50	Call	1.20649920	1.02552432	10%	10.243166%	6 months
100	Call	1.20649920	1.11601176	10%	10.243166%	6 months
200	Call	1.20649920	1.20649920	10%	10.243166%	6 months
100	Call	1.20649920	1.29698664	10%	10.243166%	6 months
50	Call	1.20649920	1.38747408	10%	10.243166%	6 months
50	Call	1.20649920	0.84454944	10%	10.243166%	one year
100	Call	1.20649920	1.02552432	10%	10.243166%	one year
200	Call	1.20649920	1.20649920	10%	10.243166%	one year
100	Call	1.20649920	1.38747408	10%	10.243166%	one year
50	Call	1.20649920	1.56844896	10%	10.243166%	one year
200	Put	1.20649920	1.19926021	10%	10.243166%	one week
400	Put	1.20649920	1.20287971	10%	10.243166%	one week
800	Put	1.20649920	1.20649920	10%	10.243166%	one week
400	Put	1.20649920	1.21011870	10%	10.243166%	one week
200	Put	1.20649920	1.21373820	10%	10.243166%	one week
200	Put	1.20649920	1.11601176	10%	10.243166%	3 months
400	Put	1.20649920	1.16125548	10%	10.243166%	3 months
800	Put	1.20649920	1.20649920	10%	10.243166%	3 months
400	Put	1.20649920	1.25174292	10%	10.243166%	3 months
200	Put	1.20649920	1.29698664	10%	10.243166%	3 months
50	Put	1.20649920	1.02552432	10%	10.243166%	6 months
100	Put	1.20649920	1.11601176	10%	10.243166%	6 months
200	Put	1.20649920	1.20649920	10%	10.243166%	6 months
100	Put	1.20649920	1.29698664	10%	10.243166%	6 months
50	Put	1.20649920	1.38747408	10%	10.243166%	6 months
50	Put	1.20649920	0.84454944	10%	10.243166%	one year
100	Put	1.20649920	1.02552432	10%	10.243166%	one year
200	Put	1.20649920	1.20649920	10%	10.243166%	one year
100	Put	1.20649920	1.38747408	10%	10.243166%	one year
50	Put	1.20649920	1.56844896	10%	10.243166%	one year

Portfolio :	**PF2**			**Currencies :**	**1-4**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	0.87828444	0.87301474	10%	11.043898%	one week
2000	Call	0.87828444	0.81241311	10%	11.043898%	3 months
500	Call	0.87828444	0.74654178	10%	11.043898%	6 months
500	Call	0.87828444	0.61479911	10%	11.043898%	one year
2000	Put	0.87828444	0.88355415	10%	11.043898%	one week
2000	Put	0.87828444	0.94415578	10%	11.043898%	3 months
500	Put	0.87828444	1.01002711	10%	11.043898%	6 months
500	Put	0.87828444	1.14176978	10%	11.043898%	one year
			Home	**currency:**	No FX	Contracts
2000	Call	0.81602930	0.81113312	10%	3.724517%	one week
2000	Call	0.81602930	0.75482710	10%	3.724517%	3 months
500	Call	0.81602930	0.69362490	10%	3.724517%	6 months
500	Call	0.81602930	0.57122051	10%	3.724517%	one year
2000	Put	0.81602930	0.82092547	10%	3.724517%	one week
2000	Put	0.81602930	0.87723149	10%	3.724517%	3 months
500	Put	0.81602930	0.93843369	10%	3.724517%	6 months
500	Put	0.81602930	1.06083808	10%	3.724517%	one year
2000	Call	0.24071504	0.23927075	10%	3.871148%	one week
2000	Call	0.24071504	0.22266141	10%	3.871148%	3 months
500	Call	0.24071504	0.20460778	10%	3.871148%	6 months
500	Call	0.24071504	0.16850053	10%	3.871148%	one year
2000	Put	0.24071504	0.24215933	10%	3.871148%	one week
2000	Put	0.24071504	0.25876867	10%	3.871148%	3 months
500	Put	0.24071504	0.27682229	10%	3.871148%	6 months
500	Put	0.24071504	0.31292955	10%	3.871148%	one year

Portfolio :	**PF2**			**Currencies :**	**5-8**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	1.87936379	1.86808761	10%	9.484678%	one week
2000	Call	1.87936379	1.73841150	10%	9.484678%	3 months
500	Call	1.87936379	1.59745922	10%	9.484678%	6 months
500	Call	1.87936379	1.31555465	10%	9.484678%	one year
2000	Put	1.87936379	1.89063997	10%	9.484678%	one week
2000	Put	1.87936379	2.02031607	10%	9.484678%	3 months
500	Put	1.87936379	2.16126836	10%	9.484678%	6 months
500	Put	1.87936379	2.44317293	10%	9.484678%	one year
2000	Call	0.00079652	0.00079174	10%	7.026373%	one week
2000	Call	0.00079652	0.00073678	10%	7.026373%	3 months
500	Call	0.00079652	0.00067704	10%	7.026373%	6 months
500	Call	0.00079652	0.00055756	10%	7.026373%	one year
2000	Put	0.00079652	0.00080130	10%	7.026373%	one week
2000	Put	0.00079652	0.00085626	10%	7.026373%	3 months
500	Put	0.00079652	0.00091599	10%	7.026373%	6 months
500	Put	0.00079652	0.00103547	10%	7.026373%	one year
2000	Call	0.01114859	0.01108170	10%	7.920480%	one week
2000	Call	0.01114859	0.01031244	10%	7.920480%	3 months
500	Call	0.01114859	0.00947630	10%	7.920480%	6 months
500	Call	0.01114859	0.00780401	10%	7.920480%	one year
2000	Put	0.01114859	0.01121548	10%	7.920480%	one week
2000	Put	0.01114859	0.01198473	10%	7.920480%	3 months
500	Put	0.01114859	0.01282088	10%	7.920480%	6 months
500	Put	0.01114859	0.01449316	10%	7.920480%	one year
2000	Call	1.20649920	1.19926021	10%	10.243166%	one week
2000	Call	1.20649920	1.11601176	10%	10.243166%	3 months
500	Call	1.20649920	1.02552432	10%	10.243166%	6 months
500	Call	1.20649920	0.84454944	10%	10.243166%	one year
2000	Put	1.20649920	1.21373820	10%	10.243166%	one week
2000	Put	1.20649920	1.29698664	10%	10.243166%	3 months
500	Put	1.20649920	1.38747408	10%	10.243166%	6 months
500	Put	1.20649920	1.56844896	10%	10.243166%	one year

Portfolio :	**PF3**			**Currencies :**	**1-4**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	0.87828444	0.88355415	10%	11.043898%	one week
2000	Call	0.87828444	0.94415578	10%	11.043898%	3 months
500	Call	0.87828444	1.01002711	10%	11.043898%	6 months
500	Call	0.87828444	1.14176978	10%	11.043898%	one year
2000	Put	0.87828444	0.87301474	10%	11.043898%	one week
2000	Put	0.87828444	0.81241311	10%	11.043898%	3 months
500	Put	0.87828444	0.74654178	10%	11.043898%	6 months
500	Put	0.87828444	0.61479911	10%	11.043898%	one year
			Home	**currency:**	No FX	Contracts
2000	Call	0.81602930	0.82092547	10%	3.724517%	one week
2000	Call	0.81602930	0.87723149	10%	3.724517%	3 months
500	Call	0.81602930	0.93843369	10%	3.724517%	6 months
500	Call	0.81602930	1.06083808	10%	3.724517%	one year
2000	Put	0.81602930	0.81113312	10%	3.724517%	one week
2000	Put	0.81602930	0.75482710	10%	3.724517%	3 months
500	Put	0.81602930	0.69362490	10%	3.724517%	6 months
500	Put	0.81602930	0.57122051	10%	3.724517%	one year
2000	Call	0.24071504	0.24215933	10%	3.871148%	one week
2000	Call	0.24071504	0.25876867	10%	3.871148%	3 months
500	Call	0.24071504	0.27682229	10%	3.871148%	6 months
500	Call	0.24071504	0.31292955	10%	3.871148%	one year
2000	Put	0.24071504	0.23927075	10%	3.871148%	one week
2000	Put	0.24071504	0.22266141	10%	3.871148%	3 months
500	Put	0.24071504	0.20460778	10%	3.871148%	6 months
500	Put	0.24071504	0.16850053	10%	3.871148%	one year

Portfolio :	PF3			Currencies :	5-8	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	1.87936379	1.89063997	10%	9.484678%	one week
2000	Call	1.87936379	2.02031607	10%	9.484678%	3 months
500	Call	1.87936379	2.16126836	10%	9.484678%	6 months
500	Call	1.87936379	2.44317293	10%	9.484678%	one year
2000	Put	1.87936379	1.86808761	10%	9.484678%	one week
2000	Put	1.87936379	1.73841150	10%	9.484678%	3 months
500	Put	1.87936379	1.59745922	10%	9.484678%	6 months
500	Put	1.87936379	1.31555465	10%	9.484678%	one year
2000	Call	0.00079652	0.00080130	10%	7.026373%	one week
2000	Call	0.00079652	0.00085626	10%	7.026373%	3 months
500	Call	0.00079652	0.00091599	10%	7.026373%	6 months
500	Call	0.00079652	0.00103547	10%	7.026373%	one year
2000	Put	0.00079652	0.00079174	10%	7.026373%	one week
2000	Put	0.00079652	0.00073678	10%	7.026373%	3 months
500	Put	0.00079652	0.00067704	10%	7.026373%	6 months
500	Put	0.00079652	0.00055756	10%	7.026373%	one year
2000	Call	0.01114859	0.01121548	10%	7.920480%	one week
2000	Call	0.01114859	0.01198473	10%	7.920480%	3 months
500	Call	0.01114859	0.01282088	10%	7.920480%	6 months
500	Call	0.01114859	0.01449316	10%	7.920480%	one year
2000	Put	0.01114859	0.01108170	10%	7.920480%	one week
2000	Put	0.01114859	0.01031244	10%	7.920480%	3 months
500	Put	0.01114859	0.00947630	10%	7.920480%	6 months
500	Put	0.01114859	0.00780401	10%	7.920480%	one year
2000	Call	1.20649920	1.21373820	10%	10.243166%	one week
2000	Call	1.20649920	1.29698664	10%	10.243166%	3 months
500	Call	1.20649920	1.38747408	10%	10.243166%	6 months
500	Call	1.20649920	1.56844896	10%	10.243166%	one year
2000	Put	1.20649920	1.19926021	10%	10.243166%	one week
2000	Put	1.20649920	1.11601176	10%	10.243166%	3 months
500	Put	1.20649920	1.02552432	10%	10.243166%	6 months
500	Put	1.20649920	0.84454944	10%	10.243166%	one year

Portfolio :	PF4			Currencies :	1-4	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	0.87828444	0.87301474	10%	11.043898%	one week
2000	Call	0.87828444	0.81241311	10%	11.043898%	3 months
500	Call	0.87828444	0.74654178	10%	11.043898%	6 months
500	Call	0.87828444	0.61479911	10%	11.043898%	one year
2000	Put	0.87828444	0.87301474	10%	11.043898%	one week
2000	Put	0.87828444	0.81241311	10%	11.043898%	3 months
500	Put	0.87828444	0.74654178	10%	11.043898%	6 months
500	Put	0.87828444	0.61479911	10%	11.043898%	one year
			Home	**currency:**	No FX	Contracts
2000	Call	0.81602930	0.81113312	10%	3.724517%	one week
2000	Call	0.81602930	0.75482710	10%	3.724517%	3 months
500	Call	0.81602930	0.69362490	10%	3.724517%	6 months
500	Call	0.81602930	0.57122051	10%	3.724517%	one year
2000	Put	0.81602930	0.81113312	10%	3.724517%	one week
2000	Put	0.81602930	0.75482710	10%	3.724517%	3 months
500	Put	0.81602930	0.69362490	10%	3.724517%	6 months
500	Put	0.81602930	0.57122051	10%	3.724517%	one year
2000	Call	0.24071504	0.23927075	10%	3.871148%	one week
2000	Call	0.24071504	0.22266141	10%	3.871148%	3 months
500	Call	0.24071504	0.20460778	10%	3.871148%	6 months
500	Call	0.24071504	0.16850053	10%	3.871148%	one year
2000	Put	0.24071504	0.23927075	10%	3.871148%	one week
2000	Put	0.24071504	0.22266141	10%	3.871148%	3 months
500	Put	0.24071504	0.20460778	10%	3.871148%	6 months
500	Put	0.24071504	0.16850053	10%	3.871148%	one year

Portfolio :	**PF4**			**Currencies :**	**5-8**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	1.87936379	1.86808761	10%	9.484678%	one week
2000	Call	1.87936379	1.73841150	10%	9.484678%	3 months
500	Call	1.87936379	1.59745922	10%	9.484678%	6 months
500	Call	1.87936379	1.31555465	10%	9.484678%	one year
2000	Put	1.87936379	1.86808761	10%	9.484678%	one week
2000	Put	1.87936379	1.73841150	10%	9.484678%	3 months
500	Put	1.87936379	1.59745922	10%	9.484678%	6 months
500	Put	1.87936379	1.31555465	10%	9.484678%	one year
2000	Call	0.00079652	0.00079174	10%	7.026373%	one week
2000	Call	0.00079652	0.00073678	10%	7.026373%	3 months
500	Call	0.00079652	0.00067704	10%	7.026373%	6 months
500	Call	0.00079652	0.00055756	10%	7.026373%	one year
2000	Put	0.00079652	0.00079174	10%	7.026373%	one week
2000	Put	0.00079652	0.00073678	10%	7.026373%	3 months
500	Put	0.00079652	0.00067704	10%	7.026373%	6 months
500	Put	0.00079652	0.00055756	10%	7.026373%	one year
2000	Call	0.01114859	0.01108170	10%	7.920480%	one week
2000	Call	0.01114859	0.01031244	10%	7.920480%	3 months
500	Call	0.01114859	0.00947630	10%	7.920480%	6 months
500	Call	0.01114859	0.00780401	10%	7.920480%	one year
2000	Put	0.01114859	0.01108170	10%	7.920480%	one week
2000	Put	0.01114859	0.01031244	10%	7.920480%	3 months
500	Put	0.01114859	0.00947630	10%	7.920480%	6 months
500	Put	0.01114859	0.00780401	10%	7.920480%	one year
2000	Call	1.20649920	1.19926021	10%	10.243166%	one week
2000	Call	1.20649920	1.11601176	10%	10.243166%	3 months
500	Call	1.20649920	1.02552432	10%	10.243166%	6 months
500	Call	1.20649920	0.84454944	10%	10.243166%	one year
2000	Put	1.20649920	1.19926021	10%	10.243166%	one week
2000	Put	1.20649920	1.11601176	10%	10.243166%	3 months
500	Put	1.20649920	1.02552432	10%	10.243166%	6 months
500	Put	1.20649920	0.84454944	10%	10.243166%	one year

Portfolio :	**PF5**			**Currencies :**	**1-4**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	0.87828444	0.88355415	10%	11.043898%	one week
2000	Call	0.87828444	0.94415578	10%	11.043898%	3 months
500	Call	0.87828444	1.01002711	10%	11.043898%	6 months
500	Call	0.87828444	1.14176978	10%	11.043898%	one year
2000	Put	0.87828444	0.88355415	10%	11.043898%	one week
2000	Put	0.87828444	0.94415578	10%	11.043898%	3 months
500	Put	0.87828444	1.01002711	10%	11.043898%	6 months
500	Put	0.87828444	1.14176978	10%	11.043898%	one year
			Home	**currency:**	No FX	Contracts
2000	Call	0.81602930	0.82092547	10%	3.724517%	one week
2000	Call	0.81602930	0.87723149	10%	3.724517%	3 months
500	Call	0.81602930	0.93843369	10%	3.724517%	6 months
500	Call	0.81602930	1.06083808	10%	3.724517%	one year
2000	Put	0.81602930	0.82092547	10%	3.724517%	one week
2000	Put	0.81602930	0.87723149	10%	3.724517%	3 months
500	Put	0.81602930	0.93843369	10%	3.724517%	6 months
500	Put	0.81602930	1.06083808	10%	3.724517%	one year
2000	Call	0.24071504	0.24215933	10%	3.871148%	one week
2000	Call	0.24071504	0.25876867	10%	3.871148%	3 months
500	Call	0.24071504	0.27682229	10%	3.871148%	6 months
500	Call	0.24071504	0.31292955	10%	3.871148%	one year
2000	Put	0.24071504	0.24215933	10%	3.871148%	one week
2000	Put	0.24071504	0.25876867	10%	3.871148%	3 months
500	Put	0.24071504	0.27682229	10%	3.871148%	6 months
500	Put	0.24071504	0.31292955	10%	3.871148%	one year

Portfolio :	**PF5**			**Currencies :**	**5-8**	
Number of FX-Contracts	Type	Price	Strike	Interest rate	Volatility	Maturity
2000	Call	1.87936379	1.89063997	10%	9.484678%	one week
2000	Call	1.87936379	2.02031607	10%	9.484678%	3 months
500	Call	1.87936379	2.16126836	10%	9.484678%	6 months
500	Call	1.87936379	2.44317293	10%	9.484678%	one year
2000	Put	1.87936379	1.89063997	10%	9.484678%	one week
2000	Put	1.87936379	2.02031607	10%	9.484678%	3 months
500	Put	1.87936379	2.16126836	10%	9.484678%	6 months
500	Put	1.87936379	2.44317293	10%	9.484678%	one year
2000	Call	0.00079652	0.00080130	10%	7.026373%	one week
2000	Call	0.00079652	0.00085626	10%	7.026373%	3 months
500	Call	0.00079652	0.00091599	10%	7.026373%	6 months
500	Call	0.00079652	0.00103547	10%	7.026373%	one year
2000	Put	0.00079652	0.00080130	10%	7.026373%	one week
2000	Put	0.00079652	0.00085626	10%	7.026373%	3 months
500	Put	0.00079652	0.00091599	10%	7.026373%	6 months
500	Put	0.00079652	0.00103547	10%	7.026373%	one year
2000	Call	0.01114859	0.01121548	10%	7.920480%	one week
2000	Call	0.01114859	0.01198473	10%	7.920480%	3 months
500	Call	0.01114859	0.01282088	10%	7.920480%	6 months
500	Call	0.01114859	0.01449316	10%	7.920480%	one year
2000	Put	0.01114859	0.01121548	10%	7.920480%	one week
2000	Put	0.01114859	0.01198473	10%	7.920480%	3 months
500	Put	0.01114859	0.01282088	10%	7.920480%	6 months
500	Put	0.01114859	0.01449316	10%	7.920480%	one year
2000	Call	1.20649920	1.21373820	10%	10.243166%	one week
2000	Call	1.20649920	1.29698664	10%	10.243166%	3 months
500	Call	1.20649920	1.38747408	10%	10.243166%	6 months
500	Call	1.20649920	1.56844896	10%	10.243166%	one year
2000	Put	1.20649920	1.21373820	10%	10.243166%	one week
2000	Put	1.20649920	1.29698664	10%	10.243166%	3 months
500	Put	1.20649920	1.38747408	10%	10.243166%	6 months
500	Put	1.20649920	1.56844896	10%	10.243166%	one year