REPORT ON THE RISKLAB PROJECT: RULES OF CAPITAL ALLOCATION AND COHERENT MEASURES OF RISK (CAPA)

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February 16, 2001

1.1 Summary.

The examination of cases mentioned by practitioners has lead to consider multiperiod risk as an important subject in studying allocation of risk capital.

It is better for a measurement of risk to take into account the historical development of the cash flows or of the market value of the final wealth. Traditional risk measures of the final worth of a position, including TailVar, fail to do this.

Using the idea behind the scenarios method, a backward recurrence scheme is used to define risk adjusted values of a future terminal position, at dates closer and closer to the one where risk capital has to be defined; the latter will be the negative of the adjusted value at this initiation date.

A specific method, which relates to the idea of price of risk, is presented and applied to several examples: time evolution of risk, comparison of various measures, discrepancy between regulatory and managerial approaches, funding liquidity.

A (still confidential) working paper *Acceptability of Multiperiod Risk* (to be commented at the RiskLab Workshop, February 28, 2001) will be available later.

1.2 On the evolution of the CAPA project.

The study of multiperiod risk measures had been announced as very first point of the project.

As mentioned above, discussions with practitioners have confirmed the importance of this step. It has proved to be a complex task on the modelling side. At this point several competing notions of acceptability are under study.

It also appeared that the role of an internal fund transfer system is very often held by the treasurer of financial institutions. While the properties of (equilibrium) pricing system are known, there is few theory about the why and how of the treasurer's function.

The rest of this report outlines the content of the attached working paper

2.1 Two unanswered questions in the case of one-period measures of risks.

So far, definitions had been given for measuring the risk of a position at the *beginning* of an holding period, the word "risk" being related to what happens at the *end* of said period. Once risk had been measured and found too important,

Key words and phrases. accounting for risks, acceptability of risk, capital allocation, coherent risk measure, decentralization, internal trading, lines of business, measure of risk, multi-period horizon, performance measurement, prospective/retrospective measures, regulators' constraints, risk-based capital, scenario, transfer pricing, treasurer, two-period problem.

no precision had been given on the remedial funding, i.e., the source of required extra capital. Moreover no description was given about the actual consequences of realisation of risk at the end of the holding period.

These two points become salient in the case where, at some intermediate date, an event either causes the need for capital or drastically perturbs the position. The former has to do with sudden non-acceptability, while the latter implies illiquidity or insolvency.

2.2 The main observation.

Most of the currently used risk measures, including the TailVar coherent measure, only depend on the *distribution* of the random variable "final net worth". While understandable for a position held for the whole observation period, this property is strongly objectionable when the history of the position, the timing of uncertainty resolution, the timing of availability of capital matter. Numerical examples (see §5 below) show this clearly.

3.1 Two possible measures of risk adjusted values.

Taking at any date-event point the minimum of conditional expected final values of a position, with respect to a set of scenarios is the obvious generalization of the one period procedure. Yet it is also possible, and in general provides a different result, to define recursively, from the paeneultimum date downwards, risk adjusted values. The interpretation of the latter is more difficult, but (see §4.1) there may be identity of the two results.

3.2 Two possible rules for acceptability.

For whatever chosen risk adjusted values system, one may require at the beginning date, either positivity of all adjusted values at date-events where the position is solvent, or simply require a positive initial risk adjusted value and keep doing the same along the way, as long as solvency is there.

4.1 Stable sets of scenarios.

It happens that both definitions of risk adjusted values coincide, when the set of scenarios is stable with respect to "pasting" pieces from scenarios at different dates or in different events at a given date.

4.2 A set of scenarios defined via price of risk concepts.

Explicit examples of stable sets of scenarios are given for binomial trees. they are very useful for computations.

5. Examples of computations.

It has been possible to examine how a supervisor and a risk manager will differ in accepting or not a project. At the first time where a decision is made, the regulator may indeed concentrate on a portfolio as it stands then, while the manager will incorporate the strategy he intends to pursue.

The importance of the way a final wealth is being reached has been shown in the case of wealths with the same distribution but different risk adjusted values.

Finally we have an example of taking care of liquidity needs with the help of borrowing or precautionary option buying.

6. Extensions.

At this stage it is necessary to pursue further the comparison between acceptance rules. This should allow for a good comprehension of allocation of capital over time and of the precise meaning of 'time diversification'.

References

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